OPTIMAL LOT SIZE POLICY FOR VENDOR-PURCHASER (S) JOINT INVENTORY CONTROL PROBLEM

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CERTIFICATE

It is to certify that the work contained in the thesis entitled "Optimal Lot Size Policy for Vendor-Purchaser(s) Joint Inventory Control Problem" by Mr. Arvind Kumar Yadav (Roll No. 9311403) has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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ABSTRACT

In a typical purchasing situation, the issue of price, lot sizing etc.., usually are settled through negotiation between the vendor and purchaser(s). Depending on the existing balance of the power, the end result of such a bargaining process may be a near optimal or optimal ordering policy for one of the parties leaving other at a disadvantage.

Recently, there has been considerable research in the area of vendor-purchaser(s) inventory control problem. In all these works, inventory control problem of vendor and purchaser(s) has been considered in an integrated manner to determine the economic lot size policy that minimizes joint total relevant cost (JTRC) of vendor and purchaser(s). Noting that coordination between vendor and purchaser(s) is a prerequisite to cope up with the emerging competition in corporate sector and for the successful implementation of new management philosophies like just in time, a system approach as an alternative approach to the vendor-purchaser(s) joint inventory control problem has been proposed.

Mathematical model has been developed for the case of single item inventory system on the basis of proposed approach and a comparison has been made with the existing models, followed by sensitivity analysis. As a step towards reality, multi-item inventory system has also been considered and analyzed. Numerical illustrations taken, suggest that the proposed approach is better than all other approaches.

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CHAPTER 1

INTRODUCTION

1.1 INVENTORY: AN OVERVIEW

Inventory problems are commonly faced by many business organizations or manufacturing industries. Whether a company buys the components and products or produces them, it is faced with decision about inventory, in particular for determining as when to procure or produce and then how much. Inventory is an idle resource held for future use. Service operations and the job shops tend to have small investments in inventory, while for many companies inventory accounts form a large percentage of the total asset value.

Inventories are neither totally good nor totally bad. Many of the problems of running out of the material are obvious. Many of the problems of having too much inventories are less obvious which is the reason why companies sometime carry more than they need. Too much inventory causes excessive holding costs, extra space requirements and may lead to product obsolescence. It hides many other problems the companies should find and solve. For instance, inventory can be used to reduce the lead time to respond to customer demand, to smooth out the production rate even when there is variation in demand, to protect the company from under estimates of demand or shortage of supply and to guard against frequent machine breakdowns etc.

Inventory basically is due to the delay in the flow of items, and it should be used only if it is cost justified. In the past, many companies tend to carry inventory to protect themselves from uncertainties instead of solving the problems related to uncertainties. They do not view inventory decisions from view point of a total system perspective. It is better to attack such problems and remove them rather than maintaining large inventories

to shield against them, which is costly. The emerging trend is to have one or two reliable suppliers who produce quality items and stick to delivery schedules. It helps in maintaining lower inventories, reduction of scrap and reworking and in lowering down inspection work. In addition companies are working to improve work method and to reduce setup costs.

1.2 NATURE OF INVENTORY RELATED COSTS

The costs associated with operating an inventory system play a major role in determining what the operating doctrine should be. The costs which influence the operating doctrine are clearly those costs which vary as the operating doctrine is changed. Fundamentally, there are five type of the costs which may be important in determining an operating doctrine

- The costs associated with procuring the units stocked.
- The costs of carrying the items in inventory.
- The costs of filling the customer orders.
- The cost associated with demand occurring when the system is out of stock.
- The cost of operating the data gathering and control procedures for the inventory system.

In the following section, we will examine in detail, each of these costs

1.2.1 PROCUREMENT COST

These costs may be divided in two parts - first there is the amount which must be paid to the source from which the procurement is made, then there are costs incurred by the inventory system itself in making a procurement This part would have no influence on operating doctrine of the system. The second part of costs arise from different factors and they can differ considerably in nature from one inventory system to another. For example, there are the costs of processing an order through the purchasing and accounting

department. These include paper and postage costs, labor costs, telephone calls bill, or the cost of computer time needed to make any necessary computation or to update accounting records. Also there are usually receiving costs incurred when the stock arrived at the warehouse. It may be necessary to uncrate the goods, perform the inspection, or perhaps carry out detailed testing. Furthermore additional accounting and control records must be prepared. These costs incurred by inventory system can be further divided in two parts - those which depend quantity ordered and those which are independent of quantity ordered. First type include transportation cost and part of inspection cost, part of receiving cost and part of inspection cost. We will find it convenient to include these costs with the cost of units themselves. Second type of cost include paper, postage, telephone charges etc.. as well as labor costs incurred in processing an order. They also include those part of receiving and inspection costs which are independent of order size. If the inventory system controls the plant where the item under consideration are made then assuming that items are made in lots, the set-up costs for a production run will fall into this category. Hence total costs of placing an order for Q units will be A + C(Q), where A is the fixed part and C(Q) is the variable part.

1.2.2 INVENTORY CARRYING COST

Included in this cost are the real out_of_pocket expenses such as cost of insurance, taxes, breakage's and pilferage at the storage site, warehouse rent if it is not owned and the cost of operating the warehouse such as light, heat, night watchman etc.

A cost which is frequently the most important cost is not the direct out_of_pocket cost but rather an opportunity cost, which is incurred by having capital tied up in inventory rather than having it invested elsewhere, and it is equal to the largest rate of return which the system could obtain from alternative investment.

Some of these cost will be proportional to the investment in the inventory (i.e. opportunity cost, breakage and pilferage costs etc..) while some costs will not depend on the investment (e.g. warehouse charge, heat, light etc..).

We will assume that inventory carrying cost incurred are proportional to the investment in inventory, although it is only an approximation.

1.2.3 COST OF FILLING CUSTOMER ORDERS

In order to fill a customer's order, a requisition must frequently be processed through some sort of accounting operation where among other things, a shipping invoice is prepared and sent to the warehouse. In the warehouse someone must go to the proper bin and obtain the unit or units. Next it may be necessary to package the order for shipment. Finally the order is shipped to the customer. After the order has been shipped, a record of this transaction is usually sent from the warehouse to the accounting where appropriate additional records are made.

The cost of the accounting operation referred to above, the salaries of those in the warehouse who are conceded with filling orders, the cost of packing, and cost of shipping, if paid by the inventory system, are all part of normal cost of filling customer's order.

1.2.4 STOCKOUT COSTS

If the customer's order arrives when the system is out of stock, it will usually be necessary to go through procedures to inform the customer of the existing situation. Two cases are possible consider first the case where all demands occurring when the system is out of stock are backordered. Backorder's cost are inherently difficult to measure since they can include such costs as loss of customer's goodwill. other part of backorder cost include the cost of notifying a customer that an item is not in stock and will be backordered plus the cost attempting to find out when the customer's order can be filled and giving him this information. Let us next consider the lost sales case. Perhaps the most

important component of the cost of lost sales is the somewhat intangible goodwill loss. This can include lost profits on sales of other item or on future sales of given item due to the fact that the customer temporarily or permanently takes his business elsewhere or he discourage other potential customers by telling them that he received unsatisfactory service. The cost of a lost sales also include the cost of any special procedure used to inform the customer that his demand cannot be supplied, and hence the profit lost in making the sale.

1.2.5 COSTS OF OPERATING THE INFORMATION PROCESSING SYSTEM

These costs may include such things as the cost associated with having a computer continuously update the inventory count or the cost of making demand prediction by records, or the cost of making an actual inventory.

1.3 PRODUCT COST

One more cost term which is not inventory related cost, but related to present work is product cost. Product cost is divided into the elemental costs.

- DIRECT MATERIAL COST The material that can be directly identified with each unit of finished product is called direct material. Material that can not be identified directly with a unit of finished product is called indirect material (e.g. cotton etc.). Cost associated with direct material is called direct material cost.
- DIRECT LABOR COST This is the labor cost which can be identified directly with a unit of finished product. for example the workers who spend their time in setting a special manufacturing set-up for a product can be identified directly with the product.
- MANUFACTURING OVERHEAD All manufacturing costs that are not direct material or direct labor are classified as overhead.

1.4 THE INVENTORY PROBLEM

The ultimate objective of any inventory problem is to answer two basic questions

i) how much to order, and

ii) when to order.

The answer of first question is expressed in terms of what is generally called as *order quantity*, it represents the optimum amount that should be ordered every time an order is placed and may vary with time depending upon the situation under consideration. The answer to second question depends on the type of inventory review policy, *periodic review* versus *continuous review*. The order quantity and the order point are normally determined by minimizing the *total relevant cost* which have been explained in previous section.

A general expression of total relevant cost is given as

Total Relevant Cost = Procurement cost (fixed and variable) + Holding cost + Shortage cost

None of the inventory model gives satisfactory solution to all the inventory problems that exist. The variation in the models is mainly due to the demand for the item which may be deterministic or probabilistic. A deterministic demand may be static, in the sense that the consumption rate remains constant with time, or dynamic, where the demand is known with certainty but varies from one time period to the other. The probabilistic demand has the two similar classifications.

1.5 PRESENT WORK

In the recent years, the problem of inventory control of vendor and purchaser (s) has got significant attention. The main focus is on minimization of sum of total relevant cost of vendor and purchaser(s), which in literature known as joint total relevant cost (JTRC), instead of minimization of total relevant cost of vendor and purchaser(s) individually. Several models have been developed to solve the joint vendor - purchaser

inventory control problem. Two important ones are (1) Joint Economic Lot Size (JELS) model by Banerjee (1986) and (2) Individually Responsible and Rational Decision (IRRD) approach by Joglekar et. al. (1990).But this type of the study (joint inventory control problem of vendor and purchaser(s)) is restricted only to single item inventory system.

In the present work, a new approach, named as System Approach has been developed to determine optimal values of inventory, ordering, manufacturing and procurement policies for a vendor and purchaser(s) considering their inventory control problem in an integrated manner under different inventory systems i.e. single item inventory system, multi-item inventory system and inventory system under *just in time* (JIT) purchasing environment.

1.6 ORGANIZATION OF THESIS

In Chapter 2, we discuss conceptual issues behind our proposed approach (System Approach) to solve vendor-purchaser inventory problem in detail and in subsequent chapters application of this approach under different purchasing scenario has been carried out. Chapter 3 deals with single item inventory system, under three different cases. Comparison of proposed model with existing models has been done algebraically as well as numerically. Chapter 4 discusses the case of multi-item inventory system. In Chapter 5, we have considered the problem under *just in time* purchasing environment. Chapter 6 presents the conclusions drawn on the basis of the present work and mentions the direction in which further work related to present one can be carried out.

CHAPTER 2

CONCEPTUAL ISSUES

2.1 REVIEW OF MODELS

The models (or alternatively, approaches) which are developed recently, as well as the traditional models, to solve the inventory control problems are summarized here. We will use these models throughout in this work and they serve the purpose of benchmarks.

2.1.1 INDEPENDENT OPTIMIZATION (IO)

As the name suggested, in this model, the inventory system (vendor or purchaser) select the operating doctrine which minimizes that system's total relevant cost, in total isolation with other inventory systems. There is no coordination required between inventory systems to operate under this model. The solution is obtained after setting the first derivative of objective function (which is either total relevant cost of vendor or purchaser) w.r.t. to deciding variable (order quantity) equal to zero and solving the resulting equation. This is one of the oldest methods, still used widely, adopted by vendor and purchaser(s). Under this model each one of them tries to optimizes himself independently and his policy depend on only his cost structure.

The Economic Lot Size (ELS) or the Economic Order Quantity (EOQ) formula attributed to Harris (1915) by Hadley and Whitin (1963) (obtained as a result of Independent Optimization) is a well known and widely used concept in purchasing and inventory management. It is the result of independent optimization adopted by any one or both parties. Since its inception, the classical ELS formula has been modified and embellished to make it applicable under a variety of conditions.

Consider for example, a typical purchasing situation where a vendor periodically produces a certain inventory item to order for a purchaser. Aside from the question of pricing, one important issue here is that of appropriate lot size. It is obvious that the purchaser's ELS for this item may not result in an optimal policy for the vendor and vice - versa.

Traditionally question of pricing, lot sizing etc. are settled through negotiation between two parties. Buffa et. al. (1979) present an excellent discussion of the basic power structure within which each negotiation take place. More often than not, depending on the existing balance of power, the outcome of such negotiation results in optimal policy for one party while the other party is subjected to substantial cost penalty; in some cases, in optimal policies results for both the parties.

2.1.2 JOINT ECONOMIC LOT SIZE (JELS) APPROACH

The term JELS was coined by Banerjee (1986). In case where the operating doctrine of one inventory system affect the other inventory system (which is always true in case of vendor and purchaser), it make sense to consider the effect of an operating doctrine on both the systems and the operating doctrine which optimizes both should be adopted. IO in most of the cases gives optimum value for one party leaving other at a great disadvantage.

Hence **this approach optimizes the total cost of vendor and purchaser taken together.** Thus in JELS, focus is on joint total relevant cost (JTRC), which is the sum of total relevant cost of vendor and purchaser, instead of individual cost of vendor and purchaser. The method of classical optimization is used to solve the problem(setting first derivative of objective function which is JTRC in this case w.r.t. deciding variables equal to zero). As Banerjee stated, the basic intent of JELS approach is " to show that a *joint*

optimal policy adopted through a spirit of cooperation can be of economic benefit to both parties."

2.1.3 INDIVIDUALLY RESPONSIBLE & RATIONAL DECISION (IRRD) APPROACH

It is quite likely that in JELS, any one or both parties provide false data to affect the operating doctrine in his own favor. The coordination required for JELS is absent in most of the industries and additional cost is required for coordination.

Joglaker et. al. (1990) proposed a new approach which they called Individually Responsible and Rational Decision(IRRD) approach. **The basic concept of this approach is to make purchaser to pay for the order handling and processing cost, they impose on vendor every time they order.** Now vendor can afford to lower his price per unit (not a quantity discount but simple price revision).The purchaser now see a lower price and consequently a lower carrying cost per unit. Faced with the added cost per order, and lower carrying cost per unit, the purchaser increases his order quantity. Joglaker et. al.(1990) showed that joint total relevant cost (total cost of vendor and purchaser together) is lower in IRRD as compared to JELS. Hence IRRD minimizes JTRC in absence of any coordinating mechanism between vendor and purchaser, and is near to the concept of *free market.*

2.2 PROPOSED MODEL : SYSTEM APPROACH

We, in this dissertation, propose a new model (approach) to solve the vendor-purchaser inventory control problem. We call this approach as SYSTEM APPROACH (SA), because conceptually, it consider the system as a whole and optimize the system cost i.e. JTRC. Vendor and purchaser(s) together constitute the system

WHAT IS SYSTEM APPROACH (SA) ?

SA is based on the following facts

1. The traditional manufacturing set up cost of vendor consists of two parts. First part is standard manufacturing set-cost cost (this consist mainly the labor cost) and second part is customer's order filling cost (mainly the order handling and processing cost). This second part of total manufacturing cost depend on ordering policy of customer (purchaser). The first part of above cost depends on -

- ordering policy of customer in the case when vendor adopt LOT-FOR-LOT manufacturing strategy.
- manufacturing policy of vendor in the case when vendor produces the item(s) in his optimal lot size quantity.

2. The vendor typically acquire the raw material and through processing converts it into finished product which is ultimately sold to the customer. The first part of above cost (standard manufacturing set-up cost) add value to the product and hence is included in the product cost. The second part of above cost (order handling and processing cost) does not add any value to the product and hence does not included in product cost. But this cost is considered before setting the selling price of the product.

3. There is a normal practice among manufacturers to keep a fixed profit margin on product cost. Let us assume that this profit margin is a ($a \geq 1$) and (Cv) is the product cost (material, labor, overhead) then the selling price of the product (Cp) will be given as

$$Cp = aCv + \text{order handling and processing cost per unit}$$

thus in system approach

- Purchaser(s) pay the order handling and processing cost incurred by vendor each time they order because this cost depend on their ordering policy.
- Purchaser(s) should also pay standard manufacturing set-up cost incurred by vendor.

Because vendor is not incurring certain costs (mentioned above), he would have otherwise incurred, he can afford a lower price per unit (not a price discount but simply a price revision). The purchaser(s) now see a lower price and consequently a lower carrying cost per unit. Faced with added cost per order and lower inventory carrying cost, a rational purchaser now increases his order size.

Because vendor is not incurring standard manufacturing set-up cost, his product cost reduces and hence his inventory carrying chargers.

We have shown algebraically (wherever possible) as well as numerically that system approach is better than both JELS and IRRD approach under different inventory systems.

2.3 RATIONALE BEHIND SYSTEM APPROACH

(1) In the past, most of the inventory models have been treated as independent subsystem without considering their relationship to other subsystem and their effect on the organization as a whole.

A general model should take into consideration the following dependence. -

- Net profit is a function of demand, price, unit cost, total inventory cost.
- The demand is a function of the selling price.
- Selling price is a function of pricing policy and of the unit cost.
- The unit cost is the function of ordering quantity.
- Total inventory cost depends on the demand, on the unit cost, and on the order quantity.

because SA consider the dependence of unit cost of the product on ordering quantity and follow a fixed pricing policy, it is more near to real world inventory problem.

(2) Few subjects in production and inventory management over the last decade have commanded such attention as just in time (JIT). The idea of reducing the raw material inventory levels to satisfy the immediate demand is appealing to all elements of the company. There is no doubt that receiving what you need in the quantities desired on a daily basis has a definite appeal. There is however another side to the JIT scenario That is the view from the supplier side. Much of the literature mention the major benefits to the supplier as being longer term contracts and the guarantee of future business. These articles stress the need for the supplier to be able guaranty the incoming material, since JIT does not allow for incoming material inspection. Today, as increasingly competitive global economy and changing production technique and shorter product life are creating the need for closer, more cooperative relationship between a firm and it's suppliers.

Making JIT work effectively is based partially upon two premises

(1) The part or component is capable of being manufactured.

(2) The design will be frozen over a sufficient period of time to justify the supplier expenditure of resource to carry out JIT.(manufacturing set up cost, normally become a expenditure because of special design of product).

Hence success of JIT depends on relationship between purchaser and buyer. Any relationship between purchaser and buyer which does not provide for " quid pro quo " will not last long. There must be strong concession to the supplier. To ask him to bear all the cost and the responsibility is dangerous. The promise of longer term contracts may not be enough to induce the supplier to adopt the JIT environment and incur the JIT costs. The transfer of order handling and processing cost and manufacturing setup cost incurred by

vendor to purchaser may provide sufficient incentive to vendor to adopt JIT purchasing conditions.

Hence System Approach discussed above fit in creating a successful JIT environment.

(3 Jogelakar et. al.(1990) contend that the coordination necessary between vendor and purchaser to implement the JELS (and hence SA)would be difficult to obtain in American Industries. They further argue that adopters of JELS would incur significant administrative costs associated with data collection and decision making that IRRD adopters would not. However, such coordination is, in fact, at work in industry by many MRP users. According to vollmen et al.(1988, pp. 209-210), modern manufacturing and control system provides the means for coordination between the vendor and purchaser. The typical manufacturing planning and control database provide information that will allow purchasers to assist vendors in determining their capacity needs, establishes a relationship that, in effect, purchases some specified capacity, determines the term of the agreement, and monitor the agreement and condition over time. Many firms now work with their vendors to plan capacity for several years into the future. Old relationships based on fear of being committed to specific firms, since suppliers, or inflexible quantities are now being replaced with an understanding of the need for mutual, ongoing relationships. The benefits of these relationships can often outweigh the expected costs of commitment, particularly under present world condition.

Hence coordination necessary for SA exists already and so its adoption is easy.

CHAPTER 3

SINGLE ITEM INVENTORY SYSTEM

3.1 INTRODUCTION

We start our discussion with relatively simple case of single item inventory system. In this chapter, we will analyze the manufacturing and ordering policy of vendor and purchaser(s) when there is only one item under consideration. We will discuss three cases under single item :

(1) Single vendor and single purchaser.

(2) Single vendor and multiple (homogenous) purchasers.

(3) Single vendor and multiple (heterogeneous) purchaser.

3.2 LITERATURE REVIEW

One pioneering work in the JELS approach is done by Monahan (1984). Restricting himself to a one manufacturer (vendor), one purchaser situation and assuming a lot-for-lot production strategy, Monahan (1984) showed that a manufacturer could develop an optimal quantity discount policy to accomplish minimization of joint cost of carrying and ordering inventory for a vendor and a purchaser. Banerjee (1986(b)) pointed out that Monahan's model needed a correction to incorporate manufacturer's inventory carrying costs. Once so corrected, Monahan's model was equivalent to Banerjee's (1986(a)) JELS model. Joglekar (1988) showed that in most realistic cases, it is not rational for the vendor to use a lot-for-lot production strategy. Lee and Rosenblatt (1986) and Goyal (1987) provided generalized versions of Monahan's model by relaxing the lot-for-lot assumption. Goyal (1988) also generalized Banerjee's JELS model by relaxing the

lot-for-lot assumption. Finally, Lee and Rosenblatt (1986), as well as, Joglekar (1988) pointed out that Monahan(1984) failed to distinguish between a manufacturer's cost of handling and processing a purchaser's order and his cost of manufacturing set up to produce a production lot. But all of above works are restricted to the one vendor, one purchaser case.

Lal and Staelin (1984), worked on the development of a quantity discount schedule for a vendor facing several groups of homogenous purchasers. Lal and Staelin's model has its own shortcomings, the most important one being that while assuming deterministically known purchaser orders, they assume that the vendors production policy will be unaffected by changes in the purchasers order quantities. Joglekar (1988) pointed out that, particularly in many purchasers situation, purchaser's order sizes affect not only the vendors revenue stream but also his manufacturing cost stream. Joglekar et. al. (1990) provides the solution of one vendor and many non identical as well as identical purchasers along with new approach which they called as individually responsible and rational decision (IRRD) approach. A comparative analysis of JELS and IRRD approach was carried by Affisco et. al. (1993).

3.3 SINGLE VENDOR - SINGLE PURCHASER INVENTORY PROBLEM

Here we restrict our discussion and analysis to a relatively simple purchasing scenario. Suppose that a purchaser periodically orders some quantity, Q of an inventory item from a vendor (supplier). With the receipt of an order, the vendor produces the required quantity of the item and on completion of the batch, ships the entire lot to the buyer.

we make following **assumptions :**

(1) vendor follows a lot-for-lot manufacturing policy.

(2) demand is static and deterministic in nature.

(3) there are no other buyers for this item and the vendor in question is the sole supplier.

(4) we consider Infinite Time Horizon.

(5) the replenishment is instantaneous.

(6)manufacturing and order lead times are known with certainty.

(7) no stock out is permitted.

Figure 3.1 shows the inventory time plots for both parties. the supply lead time, t. consists of three components: t_1 represents the time it take to transmit a purchaser order and set up a production lot, t_2 is the actual time for production, and t_3 is the time it takes to deliver the completed lot to the buyer.

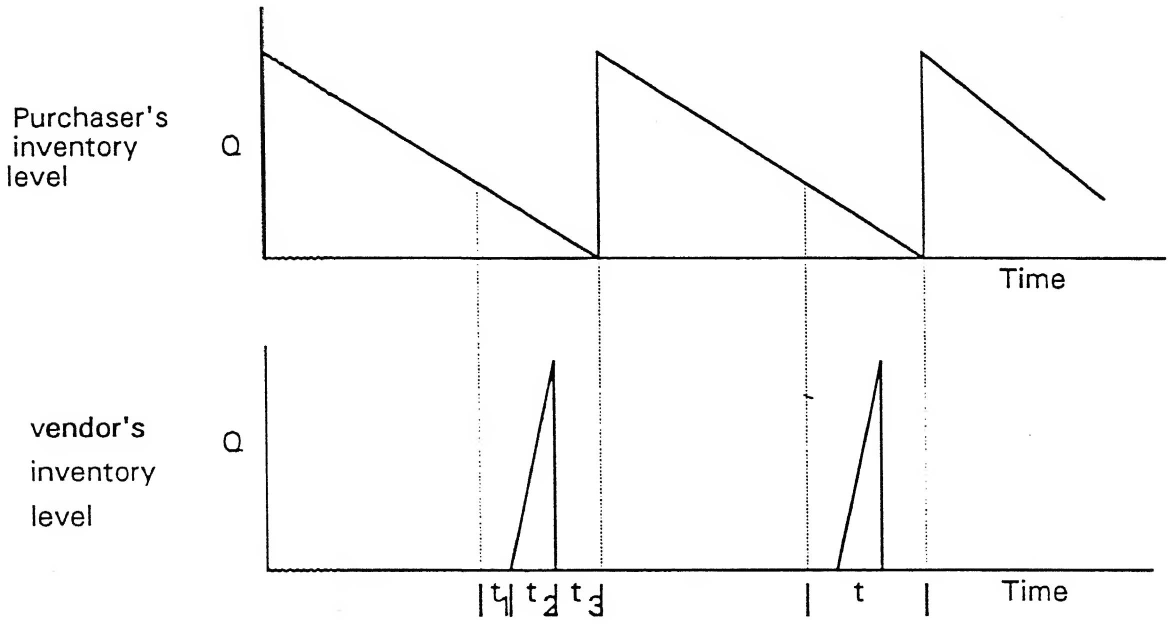


Fig 3.1

Purchaser's And Vendor's Inventory Time Plots

NOTATION USED :

D Annual demand of the item under consideration, in units

P	Vendor's annual production rate of the item, in units/year ($P \geq D$)
S_h	Vendor's order handling and processing cost per order, in Rs..
Sm	Vendor's manufacturing set up cost per production run, in Rs..
Sp	Purchaser's ordering cost per order, in Rs..
Rp	Purchaser's carrying cost per Rs. of inventory per year.
Rv	Vendor's carrying cost per Rs.. of inventory per year.
Cv	Vendor's cost of production per unit, in Rs..
Cp	Selling price of item per unit, in Rs.. ($Cp \geq Cv$)
α	Fixed profit margin on one unit of item.($\alpha \geq 1.0$)
Q	Lot size of purchaser (or vendor), in units.
TRCp(Q)	Purchaser's annual total relevant cost for any lot size Q, in Rs..
$TRC_v(Q)$	Vendor's annual total relevant cost for any lot size Q, in Rs..
JTRC	Joint total relevant cost, in Rs..

As per the explanation given in chapter 2, C_p can be expressed as

$Cp = \alpha Cv$ + Vendor's order Handling & Processing cost per unit.

RELEVANT COSTS

Only those costs which depend on the operating doctrine are considered here. These costs are called relevant costs.These are given as

PURCHASER'S COST :

Annual Ordering Cost = (D/Q)Sp

Annual Inventory Carrying Cost = (Q/2)CpRp

Purchaser's annual total relevant cost,

$$TRCp(Q) = (D/Q)Sp + (Q/2)CpRp \qquad ....(3.1)$$

VENDOR'S COST :

Annual Order Handling & Processing Cost = (D/Q)Sh

Annual manufacturing set up cost = (D/Q)Sm

Annual Inventory carrying cost = (Q/2)CvRv(D/P)

Vendor's annual total relevant cost,

$$TRC_V(Q) = (D/Q)Sh + (D/Q)Sm + (Q/2)CvRv(D/P) \quad ....(3.2)$$

Now we shall discuss the different solution methodology and a comparison of all at the end.

3.3.1 INDEPENDENT OPTIMIZATION (IO)

Two distinct possibilities under IO are given below

(1) WHEN PURCHASER OPTIMIZES

In this case purchaser optimizes his relevant cost, TRCp(Q), by setting derivative of TRCp(Q) with respect to Q equal to zero and solving it. The EOQ will be given as

$$Qp^* = \sqrt{\frac{2DSp}{RpCp}} \quad ....(3.3)$$

and the minimum total cost will be given as

$$TRCp(Qp^*) = \sqrt{2DSpRpCp} \quad ....(3.4)$$

As vendor is following a lot-for-lot manufacturing policy, his Economic Lot Size (ELS) will be same as that of Economic Order Quantity (EOQ) of purchaser, given above.

His Total Relevant Cost will be given as

$$TRCv(Qp^*) = (Sh + Sm)\sqrt{\frac{DCpRp}{2Sp}} + CvRv(D/P)\sqrt{\frac{DSp}{2CpRp}} \quad ....(3.5)$$

The joint total relevant cost (JTRC) is the sum of eqs.(3.4) and (3.5) and is given as

$$JTRC(Qp^*) = (Sh+Sm)\sqrt{\frac{DCpRp}{2Sp}} + CvRv(D/P)\sqrt{\frac{DSp}{2CpRp}} + \sqrt{2DSpCpRp} \qquad \text{........(3.6)}$$

(2) WHEN VENDOR OPTIMIZES :

In this case, the **vendor** will optimize his total annual relevant cost as given by equation (3.2) above. By setting derivative of this equation with respect to Q equal to zero, we get the ELS of vendor as :

$$Qv^* = \sqrt{\frac{Cp*RpD}{2(Sp+Sh)}} \qquad \text{.....(3.7)}$$

It is clear that in this case the total relevant cost of vendor will be minimum. This minimum TRC will be given as :

$$TRCv\ (Qv^*) = \sqrt{2DCvRv\,(Sm+Sh)D/p} \qquad \text{.........(3.8)}$$

and joint total relevant cost will be given by putting Qv* into the equation (3.1), and adding to eq.(3.8) as

$$JTRC(Qv^*) = (Sp)\sqrt{\frac{CvRv(D/P)}{2(Sm+Sh)}} + \sqrt{\frac{D(Sh+Sm)}{2CvRv(D/P)}}\,CpRp + \sqrt{2DCvRv(Sm+Sh)D/p} \qquad \text{........(3.9)}$$

Under general purchasing conditions, purchaser has the deciding hand i.e. in case of independent optimization, generally it is the purchaser who optimizes his TRC, leaving no option to vendor. *Hence in the rest of the thesis, wherever we will consider the IO, it will implies IO, when purchaser optimizes, unless otherwise mentioned.*

3.3.2 JELS MODEL

Banerjee (1986) in his paper analyzed fully the effect of IO. He calculated percentage cost penalty (PCP) of each party in the case when other party optimizes. The result of Banerjee's analysis can be summarized best in the following words :

As the difference between a and b (a represents the ratio of vendor's set up cost per set up to the purchaser's ordering cost per order, and b represents the ratio of vendor's total annual carrying cost to the purchaser's total annual carrying cost for any given lot size) grows larger, the adoption of either party's ELS places other party farther away from his own optimal position, thus increasing his cost penalty. By the same token, as the difference between a and b grows smaller, the optimal positions of the buyer and the vendor (in terms of their respective lot sizes) draw closer. when a=b (i.e. when any disparity in the two parties' fixed costs is offset exactly by a similar disparity in their carrying costs in the same direction), their optimal lot sizes are identical. Obviously, in such circumstances the adoption of one party's ELS will not result in any penalty for the other party.

Because the cost structure of vendor and purchaser is different, it is not possible to optimizes both parties simultaneously under IO model. Banerjee (1986), instead optimizes JTRC.

Hence the cost function (objective function) in case of JELS will be JTRC which is obtained by adding equation (3.1) and (3.2) and is given as :

$$\mathbf{JTRC(Q) = (D/Q)(Sp+Sh+Sm) + (Q/2)[C_p\,Rp+CvRc(D/P)]} \qquad \text{.........(3.10)}$$

by setting the first derivative of this cost function with respect to Q equal to zero, i.e

setting $\partial\,(JTRC) / \partial Q = 0$,

we get,

$$\mathbf{Qj^*} = \sqrt{\frac{2D(Sm + Sh + Sp)}{(CpRp + CvRvD/P)}} \qquad ....(3.11)$$

Substituting Qj* in eq. (3.10) above, the minimum JTRC per year is :

$$\mathbf{JTRC\ (Qj^*)} = \sqrt{2D(Sh + Sm + Sp)(CpRp + CvRvD/p)} \qquad ....(3.12)$$

Examining Equations (3.3), (3.7), (3.11) it is obvious that, Qj* lies in between Qp*and Qv*. Thus JELS represent a compromise between the buyer's ELS and the vendor's ELS when they are unequal, which usually is the case.

As JELS minimizes JTRC, JTRC(Qj*) is minimum. *Further investigation reveled that adopting a jointly optimal ordering policy, one party's loss is more than offset by the gain of the other party, and the net benefit can be shared by both parties in some equitable fashion.*

Banerjee further illustrated that appropriate discount to the party which is in loss by adopting JELS, can indeed be beneficial to both the parties.

Hence, JELS is better than IO.

3.3.3 IRRD APPROACH :

The essentials of JELS philosophy is the coordination between vendor and purchaser. It is quite likely that any one or both parties can give false data to affect the JELS in his own favor. The very concept of *free market* opposes any of such move. Hence in IRRD, purchaser optimizes his TRC after cost adjustment which has been earlier explained (purchaser pays the order handling and processing cost of vendor, each time order is placed or in other sense manufacturing set up is prepared)

We assume that in prior years the vendor and purchaser used the IO model (purchaser optimization) and that now under IRRD, the vendor lower his unit price by an amount, Cr, such that

Cr = AHv / D

Here AHv = vendor's order handling & processing cost under IO

then Cp* = The new price faced by the purchaser ;

Cp* = Cp - Cr

and

Sp* = The new ordering cost per order for the purchaser,

= Sp + Sh,

The TRC of the purchaser will be now given as

$$TRCp(Q) = (D/Q)Sp^* + (Q/2)Cp^*Rp$$

or

$$TRCp(Q) = (D/Q)(Sp+Sh) + (Q/2)C_pRp^*$$

faced with this situation the ELS of the purchaser will be (represented by Qi*)

$$\mathbf{Qi^*} = \sqrt{\frac{\mathbf{2D(Sp+Sh)}}{\mathbf{Cp*Rp}}} \qquad \text{.....}\mathbf{(3.13)}$$

and putting this value in to Eq (3.1), minimum purchaser's cost will be :

$$\mathbf{TRC(\ Qi^*)} = \sqrt{\mathbf{2D(Sp+Sh)Cp*Rp}}$$

and the JTRC will be

$$\mathbf{JTRC(\ Qi^*)} = \sqrt{\frac{\mathbf{D(Sp+Sh)}}{\mathbf{2Cp*Rp}}}\ \mathbf{CvRvD/p + Sm}\sqrt{\frac{\mathrm{Cp*RpD}}{\mathrm{2(Sp+Sh)}}} + \sqrt{\mathbf{2D(Sp+Sh)Cp*Rp}}$$

$$\text{......}\mathbf{(3.14)}$$

Here it should be noted that JTRC(Qi*) may not necessarily be less than JTRC(Qj*)

3.3.4 SYSTEM APPROACH

It seems that IRRD is conceptually better than JELS in the case when *free market* concept exist between vendor and purchaser. But it is only pseudo impression, because the very basic fact of transferring the order handling and processing cost of vendor to the purchaser is based on a feeling of cooperation, otherwise vendor may provide misleading data and can manipulate the result.

On the other hand SA is totally based on cooperation between two parties. The basic philosophy of SA has already been discussed in chapter 2. The changes in the model due to application of SA in this situation are :

Because vendor is not incurring standard manufacturing set up cost, his product cost will be lowered by an amount Cr'

where

$$\mathbf{Cr' = AMv / D}$$

(AMv = Vendor's annual manufacturing set up cost in case of *IO (purchaser optimizes)*)

hence new cost of product

$$\mathbf{Cv' = Cv - Cr'}$$

We have already mentioned that vendor is following a fixed margin policy to set the selling price of product.

selling price of the product will be :

$$\mathbf{Cp' = \alpha Cv'}$$

The SA optimizes JTRC, which in this case will be given as :

$$JTRC(Q) = D/Q(Sm+Sh+Sp) + Q/2\ (\ Cp'Rp + Cv'RvD/p\ ) \qquad ....(3.15)$$

For optimum JTRC

$$\partial\,(\,JTRC) / \partial Q = 0\ ;$$

From which we get,

$$Qs^* = \sqrt{\frac{2D(Sm + Sh + Sp)}{(Cp'Rp + Cv'RvD/P)}} \qquad ......(3.16)$$

by putting the value of Qs* in to Equation(3.15) above, the minimum value of JTRC will be :

$$JTRC(Qj^*) = \sqrt{2D(Sm + Sh + Sp)(Cp'Rp + Cv'RvD/P)} \qquad ....(3.17)$$

This is the SA solution of the problem.

3.3.5 COMPARISON :

The JTRC under basic models is given by the eqs.(3.6), (3.12), (3.14) and (3.17). It is clear that JTRC under model SA is lower than JTRC under JELS model because Cp′ is less than Cp and Cv′ is less than Cv (comparing eq.(3.12) with eq.(3.17)). Further comparing the JTRC under IO(purchaser optimizes) and IRRD,(i.e. eq.(3.6) and eq.(3.14)), one can conclude that JTRC under IRRD is lower than IO because of lower value of price in IRRD case.

There exists a value of order handling and processing cost of vendor, Sh, (for a set of values of other parameters) below which JELS performs better than IRRD and above that IRRD performs better than JELS. It is because of the fact that a higher reduction in price is obtained in IRRD when the value of Sh is high. This critical value of Sh can be find out after solving the eq.(3.12) and eq.(3.14).

System Approach provide more reductions in price as compared to IRRD and so it will always be better than IRRD.

3.3.6 AN EXAMPLE :

We are considering the same example as given in Banerjee(1986).

Consider the case of an inventory system in which an item is produced by a vendor on a lot-for-lot basis. A single purchaser periodically orders and buys a batch of this item

from the vendor who is the buyer's sole source for this item. The following parameters are known :

D = 1000 units, P= 3200 units/year

Sm = Rs. 300/setup, Sh = Sp = Rs. 100 /order

Cv = Rs. 20 /unit, Cp = Rs. 25 /unit

Rp = Rv = 0.2

RESULTS :

Results of above problem, under different models are presented here.

TABLE 3.1

EOQ/ELS UNDER DIFFERENT MODELS

MODEL TYPE	EOQ/ELS
IO(Pur. opt.)	200.00
IO(ven. opt.)	800.00
JELS	400.00
IRRD	285.71
SA	419.26

TABLE 3.2

RELEVANT COSTS OF PURCHASER UNDER DIFFERENT MODELS

model type	ordering cost	order handling cost	man. setup cost	inv. carrying cost	total
IO(Pur. opt.)	500.00	0.00	0.00	500.00	1000.00
IO(ven. opt.)	125.00	0.00	0.00	2000.00	2125.00
JELS	250.00	0.00	0.00	1000.00	1250.00
IRRD	350.00	350.00	0.00	700.00	1400.00
SA	238.51	238.51	715.53	950.16	2142.71

TABLE 3.3

RELEVANT COSTS OF VENDOR UNDER DIFFERENT MODEL

model type	order handling cost	man. setup cost	inv. carrying cost	total
IO(Pur. opt.)	500.00	1500.00	125.00	2125.00
IO(ven. opt.)	125.00	375.00	500.00	1000.00
JELS	250.00	750.00	250.00	1250.00
IRRD	0.00	1050.00	178.57	1228.57
SA	0.00	0.00	242.38	242.38

TABLE 3.4

SUMMARY

model type	JTRC	Percent Saving
IO(Pur. opt.)	3125.00	0.00
IO(ven. opt.)	3125.00	0.00
JELS	2500.00	6.25
IRRD	2628.57	4.96
SA	2385.11	7.40

RESULT ANALYSIS :

The lot size value(of vendor or purchaser) under different basic models have been shown in table 3.1. If the purchaser's ELS of 200 units is the order quantity, then the

vendor's total relevant cost is Rs.2125. In this case vendor's annual absolute cost penalty is Rs.1125, which is 112.5 % of his/her minimum total relevant cost.

On the other hand, if the vendor's ELS of 800 units is adopted, then purchaser's total relevant cost is Rs.2125, which also represent 112.5 % cost penalty for purchaser.

Thus, either party optimizes, the other party is at a significant disadvantage and resulting joint total relevant cost (JTRC) is Rs.3125.

On the other hand, JELS provide a compromise between two parties by adopting EOQ/ELS as 400 units with JTRC as Rs.2500.

It is clear from table 3.4 that JTRC under system approach is least among all models. The additional cost on purchaser, due to the adoption of SA is Rs.1142.71 while the decrease in vendor's cost is Rs.1882.62. Hence the gain of vendor is more than the loss of the purchaser and there is a net gain of Rs.739.91. This gain should be shared by both parties in some equitable fashion, like 50-50 %, or by any other measure which is acceptable to both.

3.3.7 PARAMETRIC ANALYSIS

Results of parametric analysis are summarized below

(1) Varying Sm/Sh ratio :

Fig(3.2) shows the variation in JTRC under basic models for different Sm/Sh ratio (Sh=Rs.100). It is clear from the fig that JTRC varies linearly for all models (except SA) and the difference in JTRC under SA and JTRC under IO grows larger with increase in Sm/Sh ratio. The reason for this is that when Sm is larger as compared to Sh, the reduction in the cost of product, due to transfer of this cost to purchaser under SA is more. The reduction in selling price, due to this is more and hence in JTRC.

Fig(3.3) and Fig(3.4) shows the variation of cost saving under two models (i.e. SA and IO) with respect to Sm/Sh ratio under different P/D ratio and Cp/Cv ratio

respectively. Clearly the saving is more when P/D, Cp/Cv ratios are high. Hence SA is best suited in the case of high P/D ratio along with high Cp/Cv ratio.

Fig(3.5) shows the percent saving under SA model for varying Sm/Sh ratio. A high Sm/Sh ratio, thus, implies more percent saving.

(2) Varying Sh/Sm ratio

Now we vary the cost of handling and processing an order, keeping the manufacturing setup cost as fixed. Graph is shown in Fig(3.6). From fig, one can conclude that there exists a value of Sh/Sm ratio, below which, JELS perform better than IRRD and above this value IRRD perform better than JELS. The reason for this that unless there is sufficient order handling and processing cost on vendor side, IRRD is no more better than simply IO. It is also clear from the graph that SA perform better than all other models for any value of Sh/Sm ratio. When this ratio is less, SA is near to the JELS and when this ratio is high, SA is near to IRRD.

(3) Varying other parameters

Fig(3.7) shows the variation in JTRC as linear for different models under varying Cp/Cv ratio.

Further Fig(3.8), (3.9) show the saving under SA model for different P/D ratio and R respectively. It is clear from these figures that adoption of SA is more advantageous when P/D ratio and R value is high.

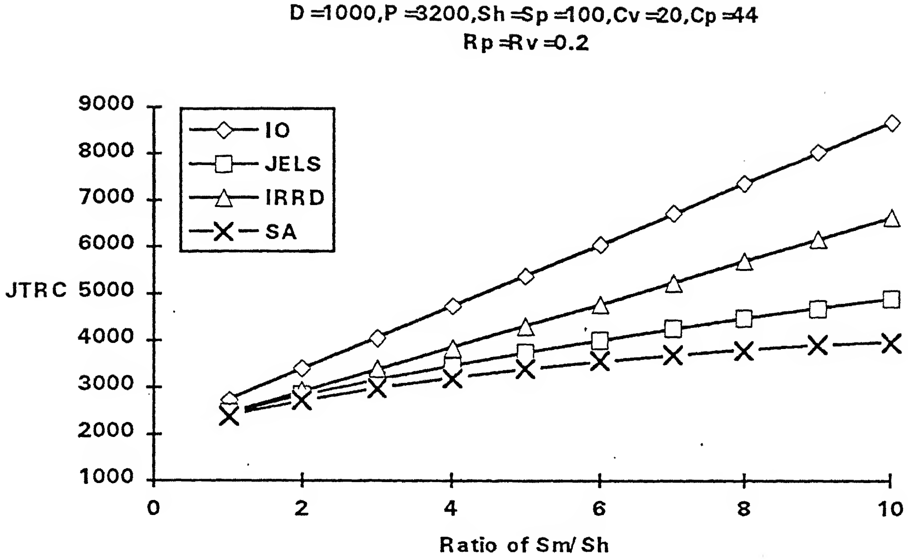


Fig 3.2
Effect of Sm/Sh ratio on JTRC

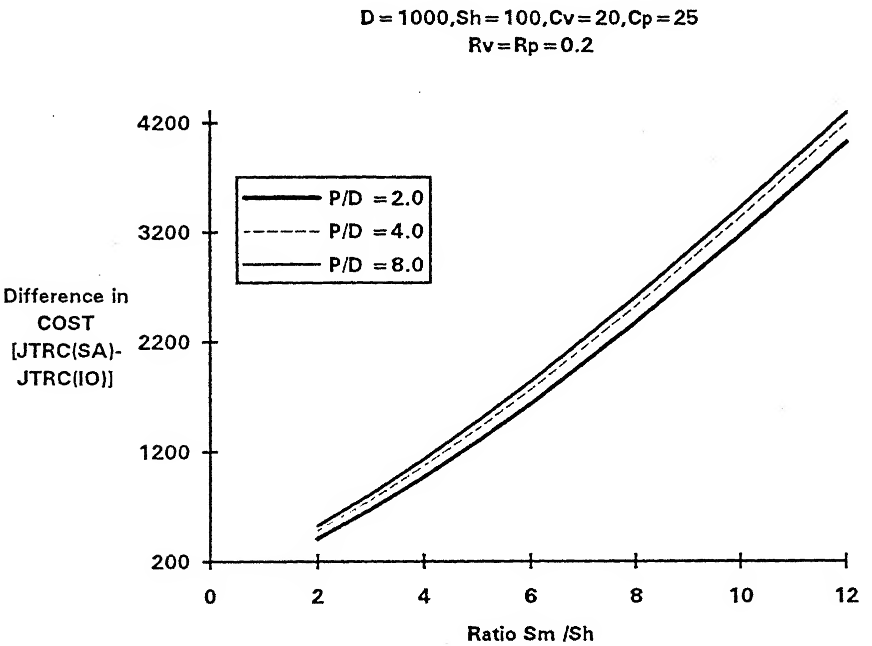


Fig 3.3
Effect of Sm/Sh ratio on saving under SA for different P/D ratio

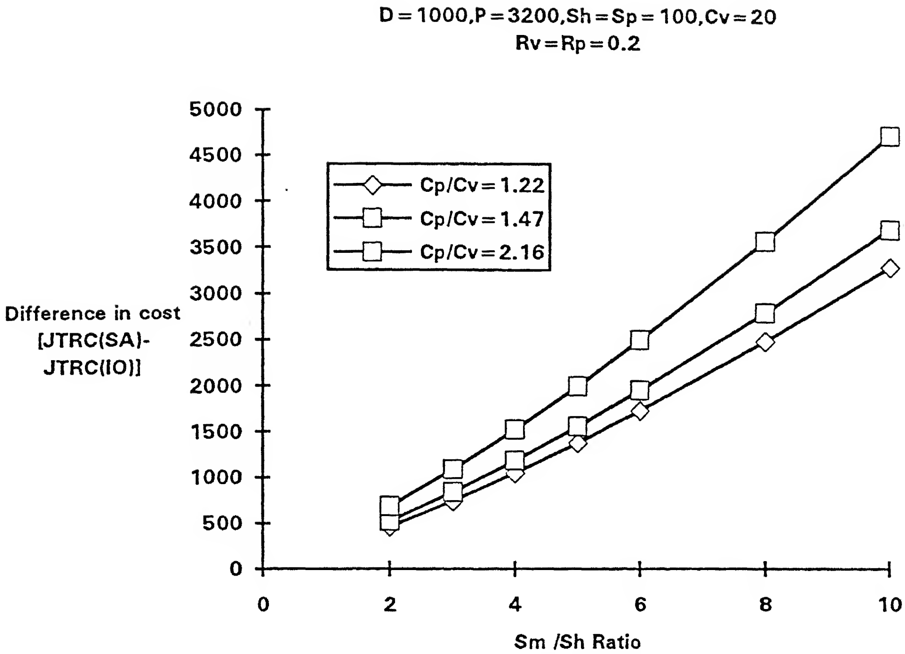


Fig 3.4
Effect of Sm/Sh ratio on saving under SA for different Cp/Cv ratio

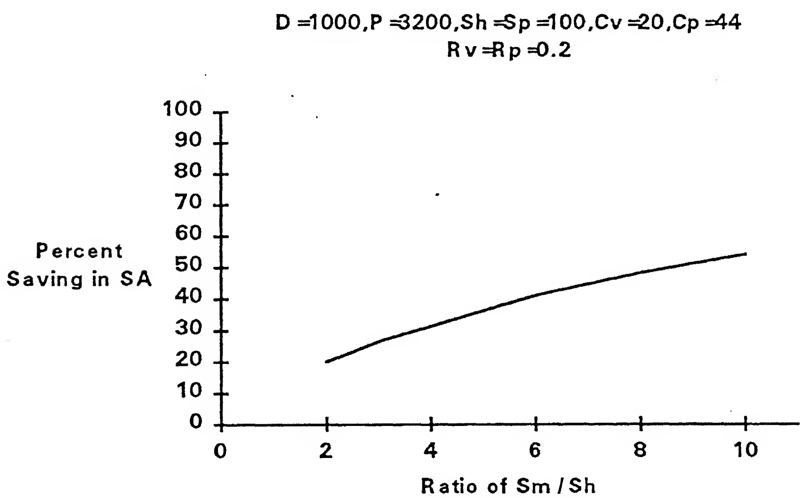


Fig 3.5
Effect of Sm/Sh ratio on percent saving under SA

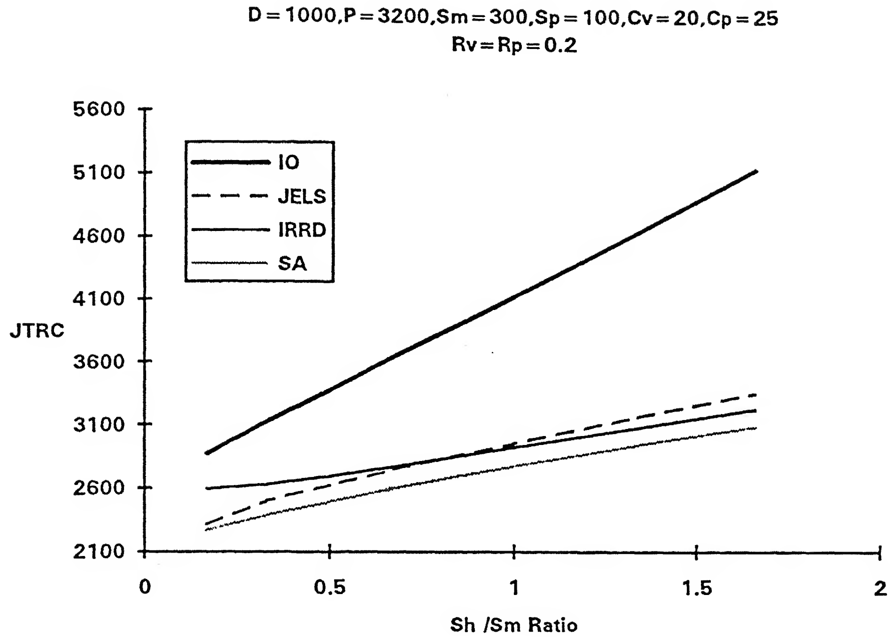


Fig 3.6
Effect of Sh/Sm ratio on JTRC

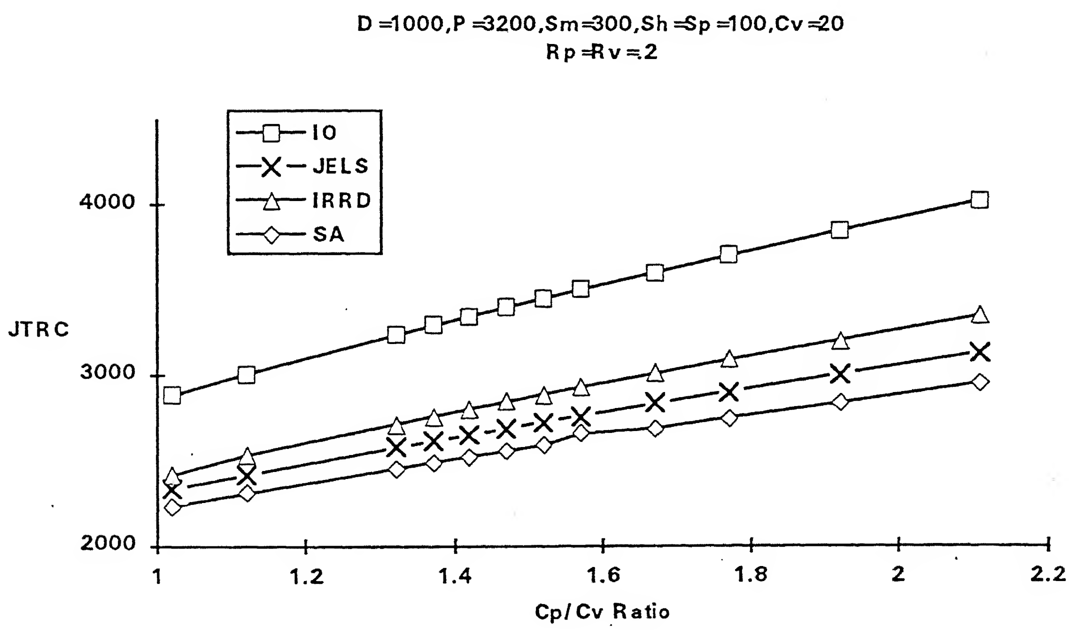


Fig 3.7
Effect of Cp/Cv ratio on JTRC

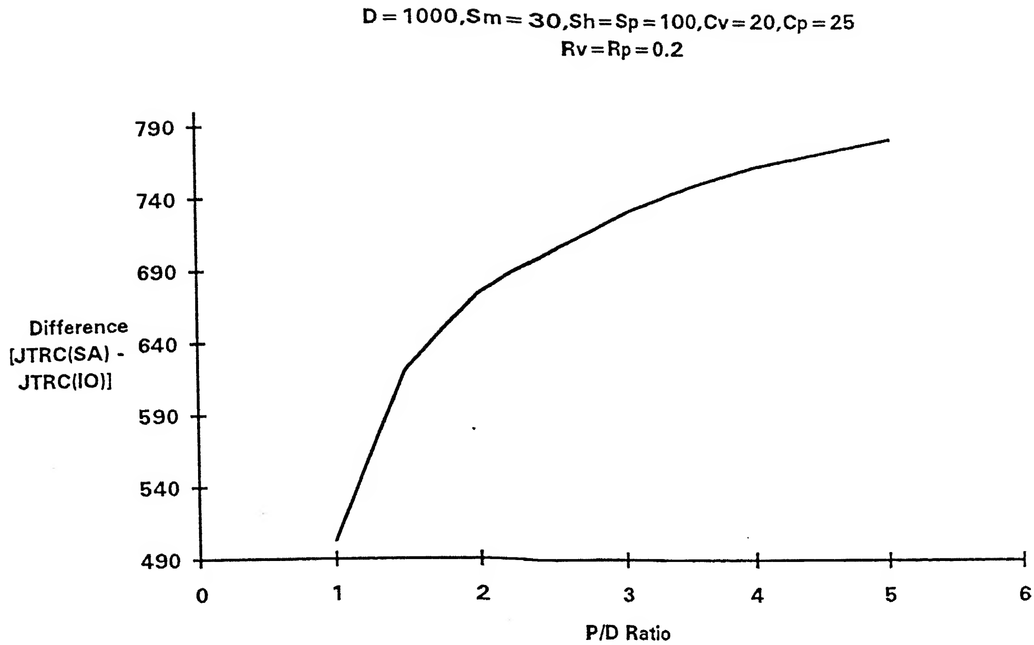


Fig 3.8
Effect of P/D ratio on JTRC

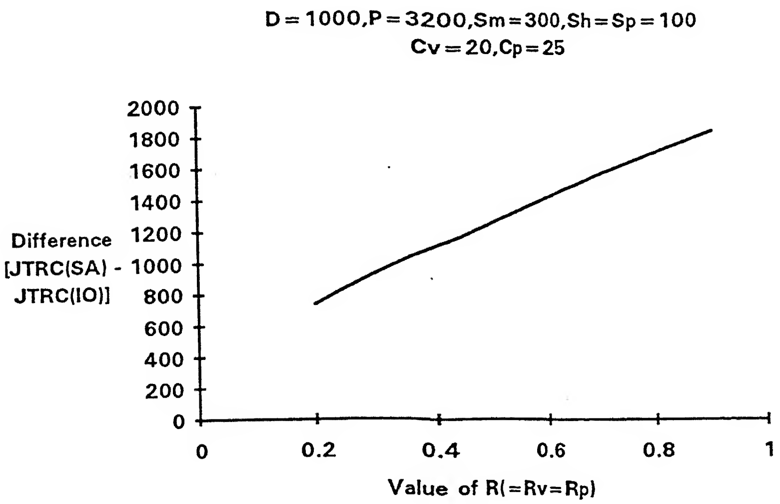


Fig 3.9
Effect of R(=Rv=Rp) on saving under SA

CHAPTER 4

MULTI - ITEM INVENTOR\

4.1 INTRODUCTION

We will now consider the problem of multi-it€ objective of minimizing the total relevant cost of vendoı Hence here we consider the case where there are a numbe: must be maintained. This is a more typical case. Very fe item which they must maintain in inventory and, as organ very decidedly in the direction of enormous numbers of iteı stock. Some organizations number their stock keeping unit:

The theoretical results derived from an analysis of system are valid in multi-item, multi-installation syste: installation can be treated as a separate entity; that is, decomposable and treated and independent single item syst independency leading to a problem decomposition is n(situation, and it become necessary to incorporate in the significant interacting factors that most naturally arise in this

One possible factor that may include interaction in system is the imposition of one or more constraints. Thı several products may be competing for a limited storage c allocation may severally constrain the aggregate level of capi entire system. Another way by which interdependency is intr

3.4 ONE VENDOR AND MULTIPLE *IDENTICAL* PURCHASERS

We consider next the case of single vendor and many identical purchasers. Consider the situation of a vendor whose annual demand D for a product comes from K ***identical*** purchasers, each contributing D/K units and ordering in same quantity.

This case is eq.uivalent to the case of one vendor and one purchaser, when vendor does not follow lot-for-lot manufacturing policy and instead manufacture the product in lot sizes so as to minimizes his total relevant cost.

We make following **assumptions**

(1) All purchasers are identical and hence order same quantity Q each time.

(2).Total demand is deterministic and static and distributed eq.ually among the purchasers.

(3).All lead times are deterministic and known in advance, so no shortages is allowed.

(4).Orders from customers are evenly paced through the year (i.e. if Q is the order quantity then two consecutive orders come exactly Q/D years apart).

(5) Vendor schedules the production of nQ[1] (n is an integer)units such that the first Q units are produced by the exact day they are to be shipped, with the remaining (n-1)Q units produced continuously thereafter over another (n-1)Q/P year(s).

(6) The planning horizon is infinite.

(7) The replenishment is instantaneous.

Although, assumption no. (4) seems unrealistically, it is quite justified for a model that advocates coordinated effort among the vendor and the purchasers under completely deterministic condition. Hence this assumption, in fact is a reality in case of System Approach.

---

[1]It is well known in inventory theory that when a vendor faces deterministically known and uniform demand , his optimal lot size is an integer multiple of the purchaser's order quantity.

NOTATION USED

We will use the notation which has been earlier used in the case of single vendor single purchaser. Following additional notation will be used

n An integer ($n \geq 1$) representing the ratio of the vendor's production lot size to purchaser's order quantity.

Q A purchaser's order quantity per order.

AOp Each purchaser's annual ordering cost.

ACp Each purchaser's annual carrying cost,

TCp Each purchaser's total annual cost of ordering and carrying the inventory.

TCk Total costs for all the k purchasers together.

AHv Vendor's annual cost of handling and processing the purchaser's orders.

ACv Vendor's annual inventory carrying cost.

AMv Vendor's annual manufacturing set up cost.

TCv Vendor's total annual cost.

JTRC Joint total relevant cost.

COSTS CALCULATION

Vendor's average annual inventory = $(Q/2)\{(n-1) -(n-2)D/P)\}$

Each purchaser's annual ordering cost

$$ACp = (Q/2)CpRp$$

Each purchaser's total annual cost of ordering and carrying the inventory

$$TCp = [D/(kQ)]Sp + (Q/2)CpRp$$

Total costs for all the k purchasers together

$$TCk = k(TCp)$$

$$= [D/Q]Sp + k(Q/2)CpRp \qquad \text{........(3.18)}$$

Vendor's annual cost of handling and processing the purchaser's orders

$$AHv = (D/Q)Sh$$

Vendor's annual inventory carrying cost

$$ACv = (Q/2)\{(n-1) - (n-2)D/P)\}CvRv$$

Vendor's annual manufacturing set up cost

$$AMv = (D/nQ)Sm$$

Vendor's total annual cost

$$\begin{aligned} TCv &= AHv + ACv + AMv \\ &= (D/Q)\,Sh + (Q/2)\{(n-1) - (n-2)D/P)\}\,CvRv + (D/nQ)Sm \quad .....(3.19) \end{aligned}$$

and JTRC will be given as

$$JTRC = D/Q\{(Sm/n) + (Sh+Sp)\} + Q/2\{[(n-1) - (n-2)D/P]CvRv + KCpRp \quad ......(3.20)$$

3.4.1 INDEPENDENT OPTIMIZATION (IO)

If each purchaser is free to choose his own Q and the vendor is free to choose his n (i.e. his optimal lot size), each purchaser 's economic order quantity (EOQp) and the optimal cost (OTCp) will be

setting $\partial\, k(TC) / \partial Q = 0$

we get

$$\mathbf{Qio^* = \sqrt{\frac{2DSp}{kCpRp}}}$$

Conseq.uently, the k purchaser's optimal total cost will be

$$\mathbf{OTCk = \sqrt{2kDSpCpRp}}$$

A rational vendor will choose an integer n* just larger or just smaller than, if not eq.ual to, the optimal value of n (n′), obtained by putting Qio* in place of Q in Eq. (3.19) and eq.uating $\partial\, TCv / \partial\, n = 0$

we get

$$n' = (1/Qio^*)\sqrt{\frac{2DSm}{(1-(D/P))(CvRv)}} \quad ....(3.21)$$

This is the IO solution to the problem. The JTRC under IO will be given by eq.uation (3.20) after putting the values of Qio* and n*.

The solution thus obtained served as benchmark against which we, compare the result of JELS, IRRD and SA.

3.4.2 JELS MODEL

Assuming a coordinated relationship between vendor and purchaser, the JELS will minimize the JTRC, which in this case is the function of Q and n, hence for JELS solution, set

$$\partial\,(\mathrm{JTRC}) / \partial Q = 0$$

and

$$\partial\,(\mathrm{JTRC}) / \partial n = 0\,;$$

these conditions give

$$Qj^* = \sqrt{\frac{2D[(Sm/n)+Sh+Sp]}{\{[(n-1)-(n-2)(D/P)]CvRv+kCpRp\}}} \quad .....(3.22)$$

and

$$n' = \sqrt{\frac{Sm\{[(2D/P)-1]CvRv+kCpRp\}}{\{(Sh+Sp)[1-(D/P)]CvRv\}}}$$

The optimal value of n, n* must be an integer. If n′ is integer, it will serve as n* and the corresponding Q* can be calculated from eq.(3.22). Otherwise, We consider each of the integer values around the calculated n′, find the corresponding Q from eq. (0 and pick the n, Q pair that minimizes the JTRC as n* and Q*.

This is generalized JELS solution to the case of one vendor and k identical purchasers.

3.4.3 IRRD APPROACH

By applying the IRRD approach in this case, the decrease in the unit price of the product will be given as

$$Cr. = AHv / D$$

(The reference is always to IO case. Hence AHv represent vendor's annual order handling and processing cost)

or, $$Cr. = Sh / Qio^*$$

the new unit price will be

$$Cp^* = Cp - Cr.$$

Using the (*) to designate all new values in IRRD of the variables in our model, we can see that

$$AOp^* = (D/kQ^*)(Sh+Sp)$$

$$AOp^* = (Q^*/2)\, Cp^*Rv$$

and $$TCp^* = (D/kQ^*)(Sh+Sp) + (Q^*/2)\, Cp^*Rv$$

which gives the optimal value of Q* as

$$Qi^* = \sqrt{\frac{\{2D(Sp+Sh)\}}{kCp^*Rp}}$$

Now TCp will be given as

$$TCp(Qi^*) = AOp(Qi^*) + ACp(Qi^*)$$

Faced with this new Qi*, the vendor recompute his n' from eq.(3.21) by using Qi* in place of Q. As in the case of IO, the best integer value of n (n*) is then found.

This is the solution of this case by IRRD approach.

3.4.4 SYSTEM APPROACH

By adopting a total coordinated approach between vendor and purchaser, as mentioned in the philosophy of SA, the solution is given as follows

The cost of production of the item under consideration will reduce by an amount Cr.′, because vendor is not incurring manufacturing set up cost.

$Cr.' = AMv / nQio^*$

(The reference is always to IO case. Hence AMv represent vendor's annual manufacturing set up cost and nQio* the lot size under IO.)

and so $Cv' = Cv - Cr.'$

Because manufacturer is following a fixed profit margin policy, the selling price of the product per unit will be given as

$$Cp' = \alpha\, Cv'$$

Hence putting the new values of Cp and Cv as CP′ and Cv′ in eq. (3.20) and solving it for optimization, we will get

$$Qs^* = \sqrt{\frac{2D[(Sm/n)+Sh+Sp]}{\{[(n-1)-(n-2)(D/P)]Cv'Rv+kCp'Rp\}}} \quad \text{......(3.23)}$$

and

$$n' = \sqrt{\frac{Sm\{[(2D/P)-1]Cv'Rv+kCp'Rp\}}{\{(Sh+Sp)[1-(D/P)]Cv'Rv\}}}$$

The optimal value of n, n* must be an integer. If n′ is integer, it will serve as n* and the corresponding Q* can be calculated from eq. (3.23). Otherwise, We consider each of the integer values around the calculated n′, find the corresponding Q from eq. (3.23) and pick the n, Q pair that minimizes the JTRC as n* and Q*.

This is SA solution to the case of one vendor and k identical purchasers.

3.4.5 COMPARISON

The algebra for comparing IO, JELS, IRRD and SA under this condition is very complex. This is because when one assume demand is known with certainty and orders are uniformly paced through the year, the vendor 's optimal solution is to produce the product in integer multiple of the purchaser's quantity. It is this discretion of the multiplier that complicate the algebra.

In the case, when we will consider a more generalized condition of one vendor and non identical purchasers in the next section, we will give a detailed comparison. However a example has been generated and the relative variation of JTRC and other parameter's is described next.

3.4.6 AN EXAMPLE

To keep consistency, we are considering the same example as in case of one vendor and one customer.

Consider the case of an item produced by a single vendor for sale to many **identical purchasers.** The demand of each customer is same.

The following parameters are known

D=1000 units P=3200units/year

Sm=Rs.300/setup Sh=Sp=Rs.100/order

Cv=Rs.20/unit Cp=Rs.25/unit

Rv=Rp=Rs.0.2/Rs.of inventory/year

Calculations have been done for the different values of K and results have been shown in Tables 3.5 to 3.10,followed by analysis.

TABLE 3.5

EOQ OF EACH PURCHASER UNDER DIFFERENT MODELS

K	each purchaser's. demand	IO	JELS	IRRD	SA
1	1000	200	400	286	344
2	500	141	224	203	230
5	200	89	126	129	131
10	100	63	91	92	95

TABLE 3.6

ELS OF VENDOR UNDER DIFFERENT MODELS

K	IO	JELS	IRRD	SA
1	400	400	572	688
2	423	448	406	460
5	445	504	516	524
10	441	455	460	475

TABLE 3.7

RELEVANT COSTS OF VENDOR UNDER DIFFERENT MODELS

K	model	setup cost	hand. cost	carrying cost	total
1	IO	750.00	500.00	400.00	1650.00
1	JELS	750.00	250.00	250.00	1250.00
1	IRRD	524.48	0.00	572.00	1096.48
1	SA	0.00	0.00	206.93	206.93
2	IO	709.22	709.22	475.87	1894.31
2	JELS	669.63	446.42	448.02	1564.07
2	IRRD	738.92	0.00	406.00	1144.92
2	SA	0.00	0.00	305.00	305.00
5	IO	674.16	1123.60	545.12	2342.88
5	JELS	595.24	793.65	598.50	1987.39
5	IRRD	581.40	0.00	612.75	1194.15
5	SA	0.00	0.00	599.70	599.70
10	IO	680.27	1587.30	559.13	2826.70
10	JELS	659.34	1098.90	557.38	2315.62
10	IRRD	652.17	0.00	563.50	1215.67
10	SA	0.00	0.00	561.92	561.92

TABLE 3.8

RELEVANT COSTS OF **EACH PURCHASER** UNDER DIFFERENT MODELS

K	model	ordering cost	carrying cost	man. set up cost	total
1	IO	500.00	500.00	0.00	1000.00
1	JELS	250.00	1000.00	0.00	1250.00
1	IRRD	699.30	700.70	0.00	1400.00
1	SA	581.39	811.15	436.00	1828.54
2	IO	354.61	352.00	0.00	707.11
2	JELS	223.21	560.00	0.00	783.21
2	IRRD	492.61	493.09	0.00	985.7
2	SA	435.00	536.82	326.08	1297.90
5	IO	224.72	222.49	0.00	447.21
5	JELS	158.73	315.00	0.00	473.73
5	IRRD	310.08	308.05	0.00	618.13
5	SA	306.04	300.61	114.76	721.41
10	IO	158.73	157.50	0.00	316.23
10	JELS	109.89	227.50	0.00	337.39
10	IRRD	217.39	215.65	0.00	433.04
10	SA	210.52	214.7	63.15	488.37

TABLE 3.9

JTRC UNDER DIFFERENT MODELS

K	IO	JELS	IRRD	SA
1	2650.00	2500	2496.48	**2035.47**
2	3308.54	3130.50	3116.32	**2900.80**
5	4578.95	4356.04	4284.80	**4206.75**
10	5989.00	5689.52	5546.07	**5445.76**

TABLE 3.10

PERCENT SAVING UNDER DIFFERENT MODELS

K	IO	JELS	IRRD	SA
1	0.00	5.66	5.79	**23.18**
2	0.00	5.38	5.80	**12.32**
5	0.00	4.86	6.42	**8.12**
10	0.00	5.00	7.39	**9.07**

RESULT ANALYSIS

From Table 3.5, it is clear that as the no. of purchasers increases, the EOQ of each purchaser decreases under all models because of the decrease in the demand of each purchaser(D/K). But EOQ, for the same K is largest under model SA because of lower values of Cv and Cp. Same is the case with ELS of vendor.

By looking Tables 3.7 and 3.8, which gives the relevant costs of vendor and purchaser, we observe that there is a decrease of Rs.1589.31 in total relevant cost of vendor under SA as compared to IO(case when K=2).On the other hand, there is a increase of Rs.1181.50 in TRC of all purchasers. Hence there exists a net gain of Rs.407.73. This gain should be shared by both parties in some eq.uitable fashion.

Also from Table 3.10, **it** is clear that percent saving under SA decreases with increase in no. of purchasers. **This is** because the demand per purchaser decreases and hence the no. of orders per year increases. In this case the overall effect of manufacturing setup cost decreases.

3.5 ONE VENDOR AND MULTIPLE *NON IDENTICAL* PURCHASERS

This is the case which **is** closest to the real life situation in case of *single item inventory system.*

All **assumptions** made earlier, in the case of one vendor and many identical purchasers, are applicable in this case also, except that the purchasers are non identical and so vendor will not produce the item in integer multiple of EOQ of any vendor.

We will use the same **notation** used in the previous cases. New notation will be defined wherever they will be used.

Let the vendor's annual demand D come from K non identical purchasers, the ith purchaser contributing D_i units of demand. Clearly

$$D_1 + D_2 + \dots\dots\dots + Di + \dots\dots + D_K = D$$

We use the subscript i to denote the variables pertaining to the ith purchaser.

Fig (3.10) shows the inventory replenishment policy of vendor and a purchaser.

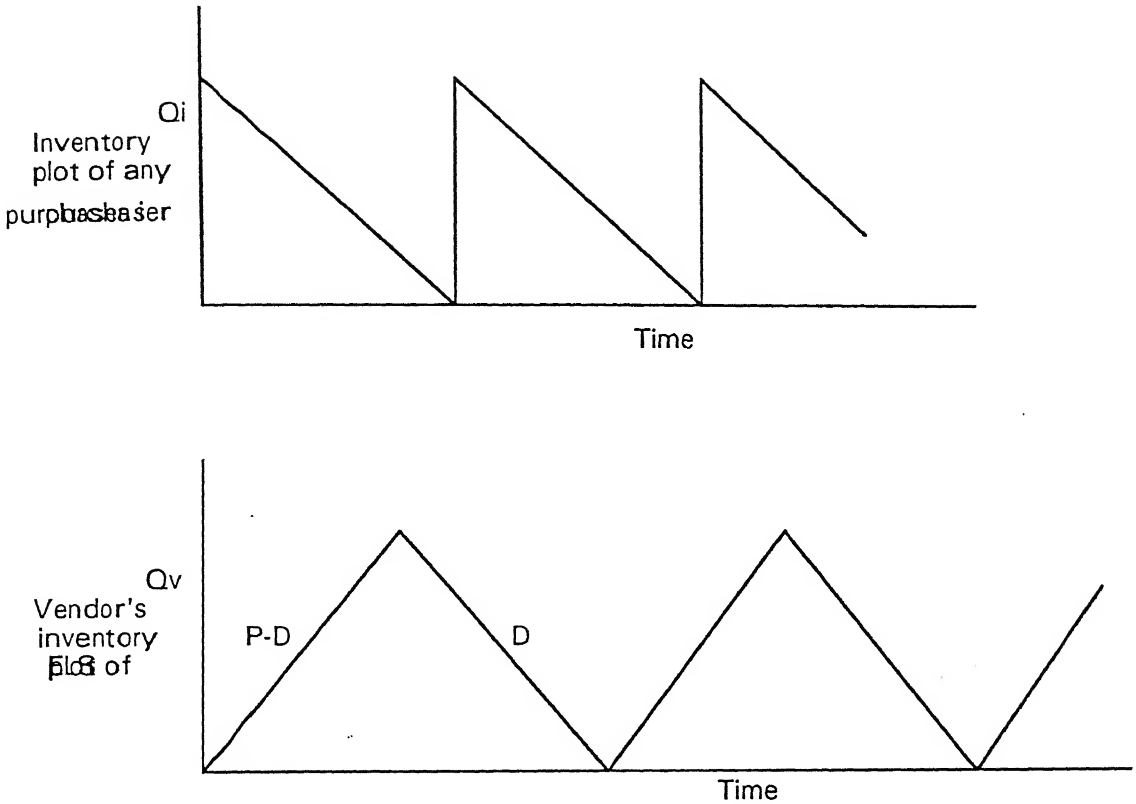


Fig 3.10
Inventory time plot of vendor and purchaser

Consider first the IO and JELS models. Here the price paid by each purchaser is Cp. Hence each purchaser's total annual cost of *ordering and carrying the inventory* (TCi)

$$TCi = (Di/Qi)Si + (Qi/2)CpRi$$

The vendor's annual cost of handling the orders from the ith purchaser will be

$$AHvi = Sh(Di/Qi)$$

The total costs of the ith purchaser (TCvi) and of the vendor to fulfill the ith purchaser's demand (not including the vendor's production set up and inventory carrying costs) are given by

$$TCvi = TCi + AHvi$$

$$= (Di/Qi)(Si+Sh) + (Qi/2)CpRi$$

Conseq.uently, KTCv, the total costs of the K purchasers and those of the vendor to fulfill their demand are

$$KTCv = \sum_{i=1}^{K} \{(Di / Qi)(Si + Sh) + (Qi / 2)CpRi\} \qquad ....(3.24)$$

Given that different purchaser's orders may be of uneq.ual quantities and that their timings may not be deterministically known, the vendor will attempt to minimize his cost of production setups and inventory carrying by using the standard production lot size model regardless of whether the lot size is an integer multiple of any of the purchaser's order quantity.[2] If Qv represents the vendor's production quantity, we know from the standard lot size model that

ACMv = vendor's annual inventory carrying and manufacturing setup cost

$$= (Qv/2)[1-(D/P)]CvRv + (D/Q)Sm \qquad ......(3.25)$$

Thus, JTRC, the joint total relevant cost of all k purchasers and the vendor (including the vendor's production setup and inventory carrying costs) is given by

$$JTRC = KTCv + ACMv \qquad ......(3.26)$$

3.5.1 INDEPENDENT OPTIMIZATION (IO)

Acting independently, the ith purchaser's optimal order quantity and corresponding total costs will be

$$Qi^* = \sqrt{(2DiSi)/(CpRi)} \qquad ....(3.27)$$

$$TCi = \sqrt{2DiSiCpRi} \qquad ....(3.28)$$

Hence,

---

[2]The vendor in this situation realizes that, given his enviornment of heterogenity of purchasers and uncertainity of order timings , he must carry some safty stock and incur the resultant overhead costs compared to the lucky vendor who deals with the identical purchasers and deterministic conditions. This is true even if the vendor is a JELS type of the master planer who can dictate to every purchaser how much to order and when. We believe that the safty stock and the resultant added costs are not significantldy diffrent under the model types, we are using ,and can be conveniently ignored in our analysis .

$$AHvi = Sh(Di/Qi^*)$$

$$= Sh\sqrt{(Di\,CpRi)/2Si}$$

$$TCvi = TCi + AHvi$$

$$= (2Si + Sh)\sqrt{DiCpRi/2Si} \qquad \text{.......(3.29)}$$

and

$$KTCv = (2Si + Sh)\sum_{i=1}^{K}\sqrt{DiCpRi/2Si} \qquad \text{.........(3.30)}$$

The vendor's optimal production and corresponding costs will be

$$Qv^* = \sqrt{\frac{2DSm}{[1-(D/P)]CvRv}} \qquad \text{.........(3.31)}$$

$$ACMv = \sqrt{2DSm[1-(D/P)]CvRv} \qquad \text{.........(3.32)}$$

Since JTRC = KTCv + ACMv under IO, the optimal system costs are given by the sum of eq.uation (3.30) and eq.uation (3.32).

3.5.2 JELS MODEL

Under JELS, we assume that a master planner decides on the optimal values of all Qi and Qv to minimize the JTRC of eq..(3.26). This is accomplished by setting

$$\partial\,(JTRC)/\partial\,Qi = 0 \qquad \text{for all i}$$

and $$\partial\,(JTRC)/\partial\,Qv = 0.$$

The condition $\partial\,(JTRC)/\partial\,Qv = 0$ results in

$$JQv^* = \sqrt{2DSm/[1-(D/P)]CvRv}$$

which is identical to the Qv of eq.. (3.31).

Thus in the general case of non identical purchasers, the vendor's production lot under JELS is the same as it was under the IO model. It follows that the vendor's annual cost of inventory carrying and manufacturing setups under JELS will be the same as the under IO given by eq.. (3.32).Thus in comparing the JTRC under the two approaches, we may safely ignore the vendor's annual inventory and manufacturing setups costs.

The condition ∂ (JTRC) / ∂ Qi = 0 gives the joint economic order quantity JQi for the ith purchaser as

$$JQi^* = \sqrt{2Di(Si + Sh) / CpRi} \qquad \text{........(3.33)}$$

Clearly the economic order quantities of eq.. (3.33) and eq.. (3.28) are quite different. It follows that the corresponding cost conseq.uences in the JELS model will also be different. To represent cost conseq.uences under the JELS model, we add the prefix J to the corresponding variables in the IO model. The costs parameter in this case will be

$$\begin{aligned} JTCi &= (Di / JQi^*)Si + (JQi^* / 2)CpRi \\ JTCvi &= (Di / JQi^*)(Si + Sh) + (JQi^* / 2)CpRi \\ &= \sqrt{(2DiCpRi)(Si + Sh)} \qquad \text{......(3.34)} \end{aligned}$$

and

$$\mathrm{JKTCv} = \sum_{i=1}^{K} (2\mathrm{DiCpRi})(\mathrm{Si} + \mathrm{Sh}) \qquad \text{......(3.35)}$$

The JTRCs under the JELS model are, of course given by the sum of eq.. (3.32) and eq..(3.35) above and is shown below

$$\mathrm{JJTRC} = \sum_{i=1}^{K} (2\mathrm{DiCpRp})(\mathrm{Si} + \mathrm{Sh})) + \sqrt{(2\mathrm{DSm})[1 - (\mathrm{D/P})]\mathrm{CvRv}} \qquad \text{.......(3.36)}$$

Since ACMv is identical in both the models, when comparing the JELS model with the IO model, we need to compare only the costs in eq..(3.35) with the eq..(3.30) or alternatively, the cost in eq..(3.34) with cost in eq..(3.29).

From eq..(3.29) and eq..(3.34), it is easy to see that TRC under the IO model will be greater than that under JELS model. To prove this, we must show that the right hand side of eq..(3.29) is greater than RHS of eq..(3.34). That is,

$$(KTCv)_{IO} > (JKTCv)_{JELS}$$

i.e. $$(2Si + Sh)\sqrt{\frac{DiCpRi}{2Si}} > (Si + Sh)\sqrt{2DiCpRi}$$

eliminating common factor and squaring

$$2(Si+Sh)^2 / 2Si \ > \ 2(Si+Sh)$$

or $$4Si^2 + Sh^2 + 4SiSh > 4Si^2 + 4SiSh$$

which is always true.

Hence JTRC in case of JELS is less than IO. ***Thus, The JELS model is clearly superior to the IO model.***

3.5.3 IRRD APPROACH

Under the IRRD model, since a purchaser must pay for the vendor's order handling costs every time he orders,

$$AOi = (Di/Qi)(Si+Sh)$$

The price per unit that each purchaser pays is also reduced from Cp to Cp′ to compensate the purchaser for the transferred order handling costs. Here it will suffice to recognize that

$$Cp' < Cp \qquad ......(3.37)$$

Now

$$ACi = (Qi/2)RiCp'$$

and

$$TCi = (Di/Qi)(Si+Sh) + (Qi/2)RiCp'$$

we uses prefix I to represent The IRRD variables.

which will be minimized when

$$IQi^* = \sqrt{\frac{2Di(Si+Sh)}{Cp'Ri}}$$

The ith purchaser's optimal ordering and carrying costs will be given by

$$ITCi = \sqrt{2Di(Si+Sh)Cp'Ri} \qquad ....(3.38)$$

Under the IRRD model, the vendor has no order processing and handling costs. Of course, the vendor's revenue from the ith purchaser is also reduced by an amount Di(Cp'-Cp) but that need not be accounted for in the systems costs, since it is simply a transfer cost. As we argued before, the vendor's production lot size and the corresponding costs of manufacturing setups or related inventory carrying costs will be identical to those under the two models, thus, we need not to count for them either. Hence, to compare the system costs under JELS with those under IRRD, We simply have to compare the cost in eq..(3.37) with eq..(3.34). From eq..(3.36), it is clear that the RHS of eq..(3.37) is less than RHS of eq..(3.34)

The JTRC, will be given by

$$IJTRC = \sum_{i=1}^{k}(2DiCp'Rp)(Si+Sh)) + \sqrt{(2DSm)[1-(D/P)]CvRv} \qquad ....(3.39)$$

Hence, we have proved that the system costs under the IRRD model will be less than those under the JELS model.

3.5.4 SYSTEM APPROACH

Under the SA, a purchaser pays for the vendor's order handling and processing cost like IRRD model. In addition to this, all purchasers together pay for the vendor's manufacturing setup cost. This could be an incentive given to the vendor, as discussed in chapter 2. Any suiTable measure for the distribution of this cost among the purchasers

may be adopted,which should be fair and accepTable to all. One such measure could be the demand of each individuals.

The reduction in the cost of manufacturing a item will be

$$Cv'' = Cv - Sm / Qv$$

and the corresponding selling price will be

$$Cp'' = \alpha Cv''$$

(here α is the profit margin on product, which remain the same)

The cost transferred to the purchasers side under SA is more than IRRD and so the price reduction under SA will be more than under IRRD i.e.

$$Cp'' < Cp' \qquad ....(3.40)$$

The different values of variables under SA will be given as (*we will use the prefix S to represent the variable's value under SA*)

$$SQv* = \sqrt{\frac{2DSm}{[1-(D/P)]Cv''Rv}}$$

and the vendor's annual inventory carrying cost

$$ACv = (SQv*/2)\ Cv''Rv$$

The EOQ of ith purchaser will be given as

$$SQi* = \sqrt{\frac{2Di(Si+Sh)}{Cp''Rp}} \qquad ....(3.41)$$

and the ith purchasers total ordering and inventory carrying cost will be

$$STCi = \sqrt{2DiCp''Ri(Si+Sh)}$$

total cost of ordering and inventory carrying for all K purchasers will be

$$SKTCi = \sum_{i=1}^{k}(2DiCp''Ri)(Si+Sh) \qquad ....(3.42)$$

In addition to this the manufacturing setup cost paid by all purchasers together will be

$$MC = (D/SQv*)\ Sm \qquad ....(3.43)$$

The JTRC will be the sum of eq.s.(3.42), (3.32) and will be given as

$$SJTRC = \sum_{i=1}^{k}(2DiCp''Rp)(Si + Sh) + \sqrt{(2DSm)[1-(D/P)Cv''Rv} \quad ....(4.44)$$

Comparing the JTRC in SA model with that in JELS and IRRD {i.e. comparing eq..(3.44) with eq.s.(3.36), (3.39)},with the help of eq..(3.40), we can say that SA is better than both.

Hence Joint Total Relevant Cost under SA is minimum.

3.5.5 AN EXAMPLE

To maintain consistency, we are considering the same example as, we have considered in case of identical purchasers.

Consider the case of an item produced by a single vendor for sale to many non identical purchasers. The total annual demand on the vendor is simply the sum of the different annual demands of the purchasers.

The following parameters are known

D=1000 units	P=3200units/year
Sm=$300/setup	Sh=Sp=$100/order
Cv=$20/unit	Cp=$25/unit
Rv=Rp=$0.2/$of inventory/year	

Calculations have been done for the different values of K and results have been shown in Tables 3.11 to 3.16,followed by analysis.

RESULTS

Results of above problem have been given in the following Tables.

TABLE 3.11

`ELS OF VENDOR UNDER DIFFERENT MODELS

K	IO	JELS	IRRD	SA
2	467.10	467.10	467.10	474.75
5	467.10	467.10	467.10	474.75
10	467.10	467.10	467.10	474.75

TABLE 3.12

EOQs OF PURCHASERS UNDER DIFFERENT MODELS

K	purchaser number	demand	IO	JELS	IRRD	SA
2	1	750	173.21	244.95	248.25	252.45
	2	250	100.00	141.42	143.38	145.75
5	1	400	126.49	178.89	182.76	185.85
	2	250	100.00	141.42	144.49	146.90
	3	200	89.44	126.49	129.23	131.41
	4	100	63.25	89.44	91.38	92.92
	5	50	44.72	63.25	64.62	65.70

10	1	200	89.44	126.49	130.47	132.63
	2	175	83.67	118.32	122.04	124.07
	3	150	77.46	109.52	112.04	114.86
	4	125	70.71	100.00	103.14	104.86
	5	100	63.25	89.44	92.25	93.78
	6	75	54.77	77.46	79.89	81.22
	7	67	51.77	73.21	75.51	76.77
	8	50	44.72	63.25	65.23	66.31
	9	33	36.33	51.38	53.00	53.81
	10	25	31.62	44.72	46.13	46.89

TABLE 3.13

RELEVANT COSTS OF VENDOR FOR DIFFERENT MODELS

K	model	setup cost	hand. cost	carrying cost	total
2	IO	642.26	683.00	642.26	1967.52
2	JELS	642.26	482.26	642.26	1767.48
2	IRRD	642.26	0.00	642.26	1284.52
2	SA	0.00	0.00	631.86	631.86
5	IO	642.26	1059.75	642.26	2344.77
5	JELS	642.26	749.36	642.26	2033.88
5	IRRD	642.26	0.00	642.26	1284.52
5	SA	0.00	0.00	631.86	631.86

10	IO	642.26	1509.36	642.26	2793.88
10	JELS	642.26	1067.27	642.26	2351.79
10	IRRD	642.26	0.00	642.26	1284.52
10	SA	0.00	0.00	631.86	631.86

TABLE 3.14

RELEVANT COSTS OF ALL PURCHASERS UNDER DIFFERENT MODELS

K	model	ordering cost	carrying cost	man. set up cost	total
2	IO	683.00	683.00	0.00	1366.00
2	JELS	482.93	965.93	0.00	1448.89
2	IRRD	952.70	952.70	0.00	1905.40
2	SA	937.21	937.21	631.86	2506.30
5	IO	1059.75	1059.75	0.00	2119.50
5	JELS	749.36	1498.72	0.00	2248.08
5	IRRD	1466.90	1466.90	0.00	2933.81
5	SA	1442.65	1442.33	631.9	3516.9
10	IO	1509.37	1509.37	0.00	3018.74
10	JELS	1067.27	2134.54	0.00	3201.81
10	IRRD	2069.52	2069.52	0.00	4139.04
10	SA	2035.62	2035.62	631.86	4703.10

TABLE 3.15

JTRC UNDER DIFFERENT MODELS

K	IO	JELS	IRRD	SA
2	3333.52	3216.37	3189.92	3138.16
5	4463.77	4281.96	4218.33	4147.00
10	5812.62	5553.60	5432.56	5339.96

TABLE 4.16

PERCENT SAVING UNDER DIFFERENT MODELS

K	IO	JELS	IRRD	SA
2	0.0	3.51	4.31	**5.86**
5	0.0	4.07	5.51	**7.04**
10	0.0	4.45	6.70	**8.21**

RESULT ANALYSIS

Table (3.11) presents the lot size of vendor for four lot sizing policies - IO, SELS, IRRD, SA for a set of three different values of K. Table (3.12) presents the economic ordering quantities of purchaser results. For all four models, the vendors economic order quantity depend on total demands and is independent of how this demand is distributed among the purchase. Lot sizes are largest for SA and smallest for IO. This is due to the fact that handling cost in part determines the lot sizes while this is not true in case of independent optimization. As predicted, the JTRC for the SA model are less than IRRD, JELS and IO. On a prop. basis, the savings over the IO approach increases for all 3 models with increasing K (as shown in Table (4.16)).

The SA policy also creates an imbalance in benefits to the parties. In all cases, the vendor realizes a reduction in costs, compared with IO, while total purchasers costs increases. For our illustration, these values are \$1335.66 and \$1140.3 for K=2, \$1712.91 and \$1397.4 for K=5, \$2162.02 and \$1684.36 for K=10. Therefore, negotiation between the parties is necessary to assure that the net benefit can be shared in some eq.uiTable fashion. Accordingly to one policy that may be simply implemented, the vendor can offer the purchasers a one time yearly rebate sufficient to recover their increased costs plus an additional amount eq.ual to 50% of the remaining vendors net gain. Thus, in case of two purchasers, they would receive \$1237.98 and vendor \$97.68, the five purchasers would receive \$1555.15 and vendor \$157.75, ten purchasers would receive \$1923.19 and vendor \$28.83. The distribution of net profit among the purchasers should be in proportion to their demand, or in the proportion to their payment of manufacturing set up cost.

3.5.6 PARAMETRIC ANALYSIS

The variation in JTRC under basic models, and saving in the case of SA model with respect to the problem parameters are discussed in this section.

(1) Varying manufacturing setup cost(Sm) of vendor

Manufacturing setup cost of vendor is included in the cost of the production. Under SA, when this cost is transferred to the purchasers side, a reduction in cost of product (Cv), and hence in the selling price of the product (Cp) is obtained. A higher value of Sm means larger reduction in the values of Cv and Cp. It is clear from the eq.s. derived, that JTRC depend on the Cv and Cp values. A lower set of these values gives lower value of JTRC. Hence under SA, the JTRC will be minimum because Cv and Cp are minimum, keeping other parameters constant. This is shown graphically in Fig(3.11)

Further, with higher manufacturing setup cost, higher saving is achieved under model SA. Fig(3.12) shows the variation in saving under SA with varying Sm. Clearly saving is more with higher values of Sm.

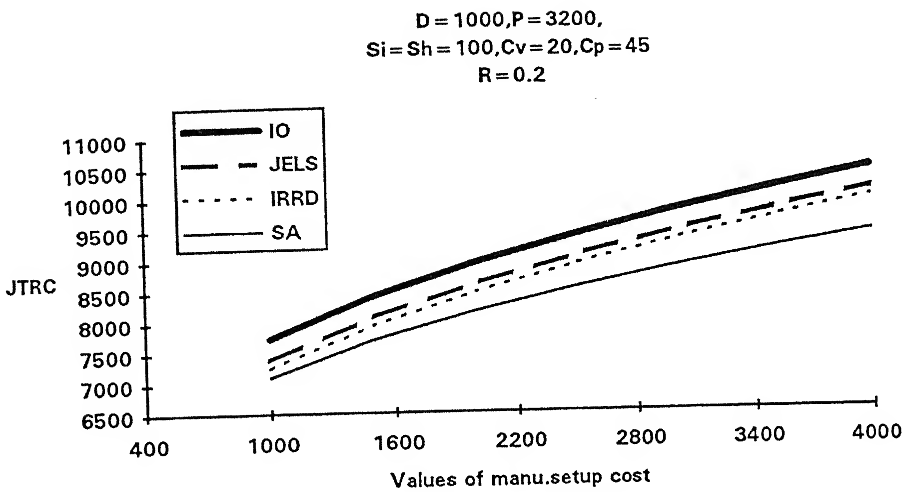


Fig 3.11
Effect of manu. setup cost on JTRC

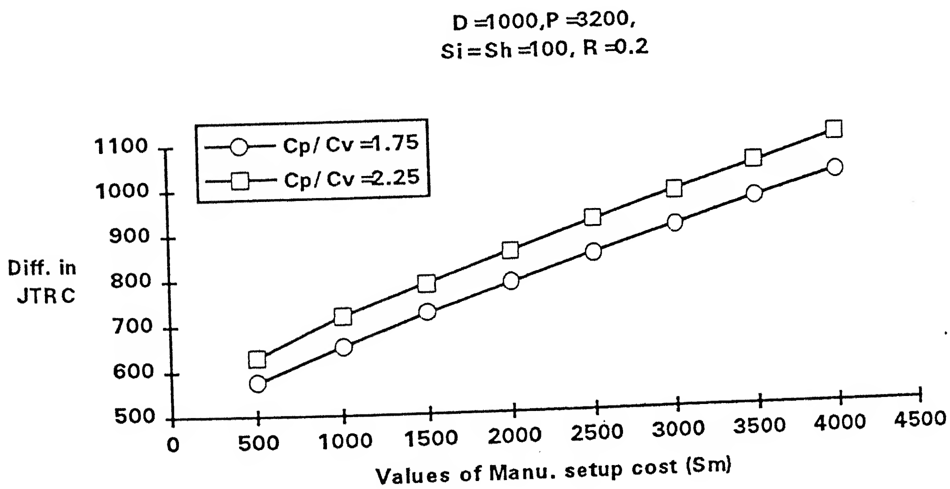


Fig 3.12

Effect of manu. setup cost on saving under SA

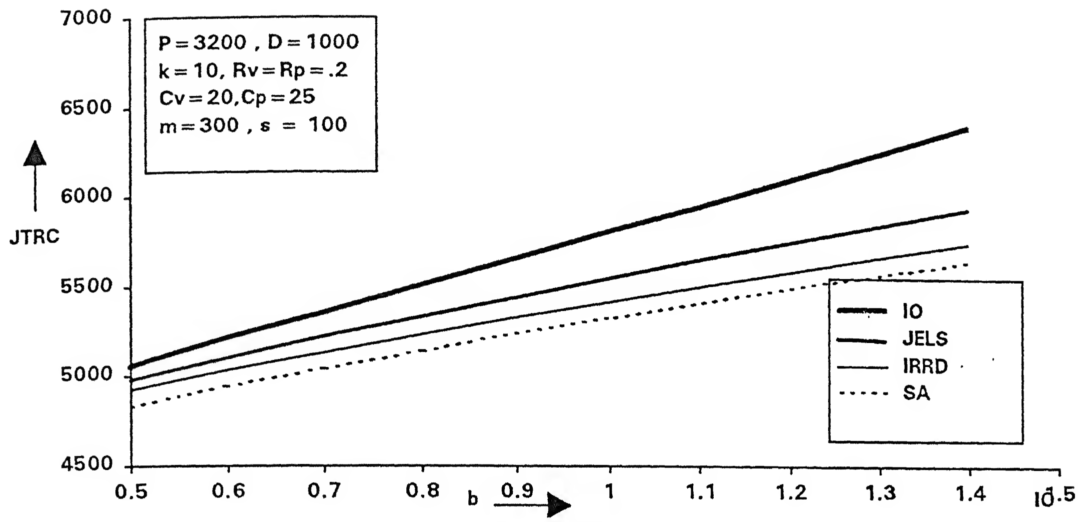


Fig 3.13
Effect of b on JTRC

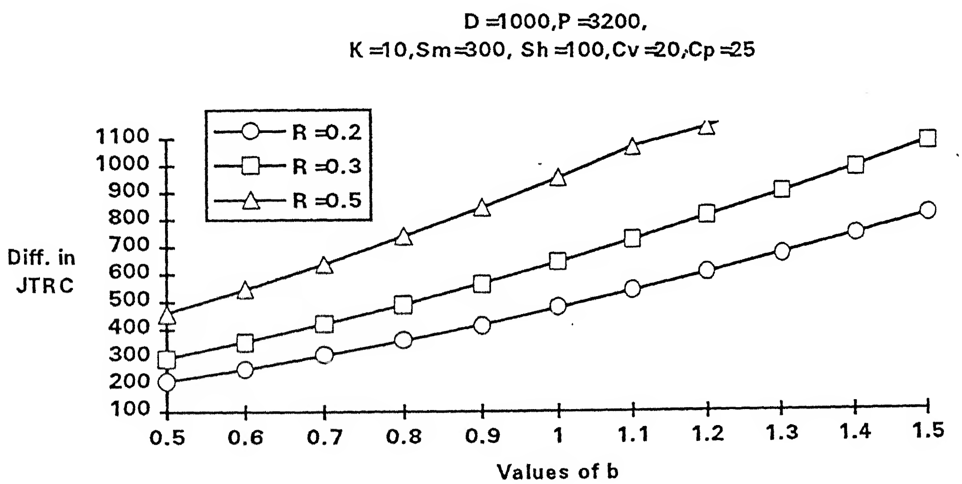


Fig 3.14
Effect of b on saving under SA

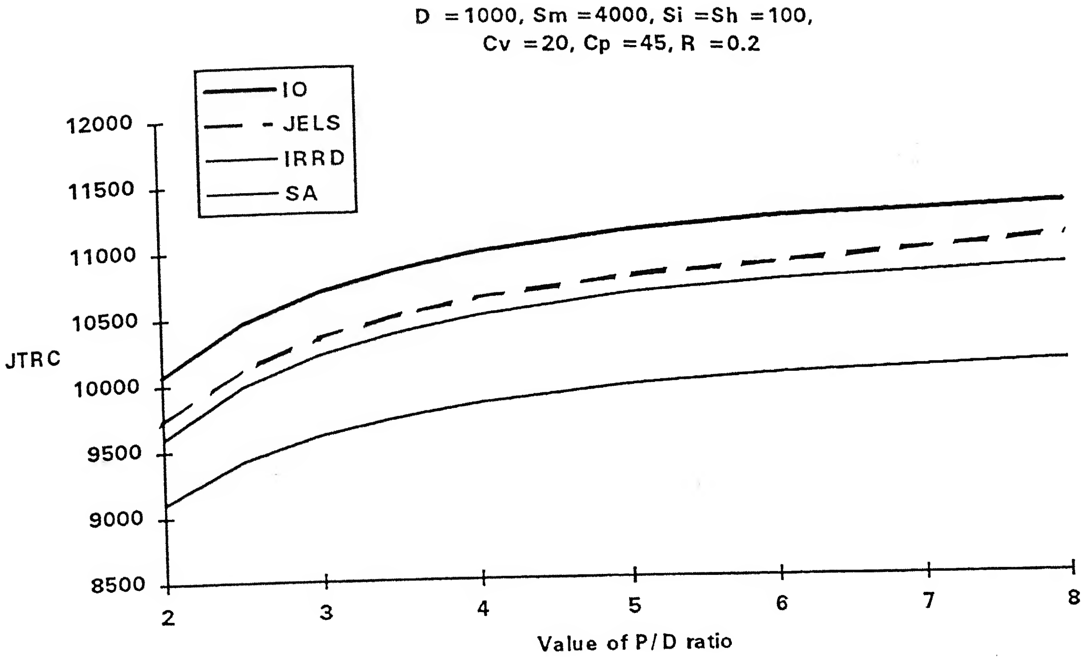


Fig 3.15
Effect of P/D ratio on JTRC

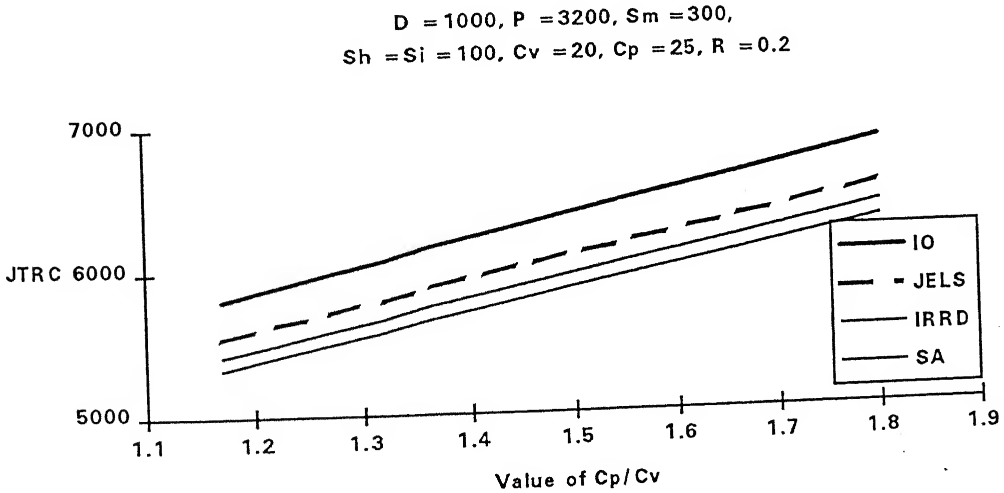


Fig 3.16
Effect of Cp/Cv ratio on JTRC

production of commodities, and, more specifically, in the set up or order cost. We shall consider this type of interdependency for our problem in this chapter.

Multi-item, multi-installation systems have the following three simplest structure

(a) Multi-item, single installation system.

(b) Single item, multi-installation parallel system.

(c) Single item, multi-installation series system.

We will consider the simple case of Multi-item, Single installation system.

GENERAL NATURE OF PROBLEM

When a product is packaged in more than one type of container immediately after manufacture, these items (each representing a particular type or size of container) are said to be *jointly replenished.* There is saving in the cost if these items are manufactured jointly and then packaged individually. The economic advantage of manufacturing these items jointly and then packaging them individually is due to the fact that if items are replenished (manufactured and packaged) individually then each item is accountable for the full set up cost for each of its manufacturing and packaging runs. However, a policy of policy of packaging all the items whenever a manufacturing set up is undertaken can be uneconomic because it may be economical to package certain items less frequently than others. In joint replenishment inventory system, hence, the problem is essentially to determine the optimum packaging frequency of individual items. The item with the greatest packaging frequency, however, provides the optimum manufacturing frequency of the unpackaged product.

The problem stated above is same for a group of items which share some manufacturing processes which is common to all, and some manufacturing processes which are exclusively for a particular item.

It may be stated further that the problems stated above are akin to the problem of determining economic order quantities for items furnished by a single supplier. Here, the cost of placing a purchase order for a number of items consists of two components

- A fixed cost which is independent of the number of items ordered.
- A variable cost which dependents on the items being ordered.

The first component of the ordering cost in the multi-item single supplier inventory system is akin to the manufacturing set up cost for the product in the joint replenishment inventory systems and the second element of the ordering cost in the multi-item single supplier system is similar to the individual packaging set up cost for each item.

4.2 STATEMENT OF PROBLEM

Here we assume that there exist a group of items, which have major manufacturing process common (i.e. same product, minor changes in shape, design or dimensions of individual item) with some specialized manufacturing processes associated with each item.

There exists a major manufacturing set up cost which is independent of the number of the items being manufactured in a lot, and minor manufacturing set up cost which occurred due to inclusion of any item in the manufacturing lot. Looking from the purchaser's side, there will be a major ordering cost independent of the number of the items ordered at a time and minor ordering cost associated with the inclusion of any item in the ordering lot. As, we have separated out order handling and processing cost from traditional manufacturing set up cost, there will be a major order handling and processing cost and minor order handling and processing cost for each item.

Here the problem is to find out optimum manufacturing policy of items (or equivalently, optimum ordering policy of the items) in the case of *single supplier-single purchaser*. We assume that supplier is the only producer of the items and purchaser, the only customer of these items.

We will analyze the problem from I.O, JELS, IRRD and SA methodology, as done earlier.

4.3 LITERATURE REVIEW

In the past, there has been a considerable attempt to study the multi-item inventory system. Each has discussed the problem under appropriate operating policy. Iterative solution procedures have been given by a number of authors namely Brown (1967), Shu (1971), Doll and Whybark (1973) and Goyal (1973,1974(a),1974(b)). Nucturne (1973) has given a graphical solution for the special case of two items. Silver (1976) has suggested a simpler non - iterative approach.

In a study, Kapsi and Rosenblatt (1985) suggested combining the Silver's (1976) and Goyal's (1974(b)) heuristics. The first trial value of N is obtained using Silver's method and then Goyal's heuristic is implemented. The combined approach consistently performed better in simulation experiments.

In addition to these heuristic methods, a number of other methods, which are essentially variations of these methods, have appeared in literature. Jackson et al. (1985) restricted the Ki values to powers of 2.Therefore Ki is restricted to be 1,2,4..... A similar idea was used earlier by Goyal (1975(b)) and Hassler and Hogue (1976) in scheduling problem.

In general, near optimal solution can be obtained by using the heuristics of Kapsi and Rosenblatt (1985) or Goyal (1974(b)). We will use the heuristic proposed by Goyal (1974(b)) to solve the multi-item inventory problem.

4.4 MATHEMATICAL MODEL AND SOLUTION METHODOLOGY

ASSUMPTIONS

Following assumptions are being made

(1) Single supplier and single purchaser existence..

(2) As supplier is the only party who produces the items and purchaser the only customer, supplier follow lot - for -lot manufacturing policy.

(3) Lead time for procuring the supplies is constant.

(4) Stock outs are not permitted.

(5) Minimization of cost is taken as the criterion of optimality.

(6) Time horizon is infinite.

(7) The purchase orders are placed at equal time intervals.

(8) An item is replenished at equal time intervals.

(9) The demand rate of each item is constant and deterministic.

NOTATION USED

n	Total number of items.
i	Item number (i = 1,2.....n).
Di	Demand of item no. i, in units/year.
Qi	Replenishment quantity of item i in units.
M	Major manufacturing set up cost in Rs.
S	Major order handling and processing cost of vendor in Rs.
Mi	Minor manufacturing set up cost for item i in Rs.
Si	Minor order handling and processing cost for item i in Rs.
A	Major order placing cost of purchaser in Rs.
Ai	Minor order placing cost for item i in Rs.
Rvi	Inventory carrying charge of vendor, for item i, in Rs./Rs./year.
Rpi	Inventory carrying charge of purchaser, for item i, in Rs./Rs./year.
Cvi	Cost of producing a unit of item i, in Rs.
Cpi	Selling price of one unit of item i, in Rs.
αi	Profit margin on item i. (i.e.. αi = Cpi / Cvi)
μ	A factor, showing the effect of profit margin.

βi Ratio of vendor's minor order handling and processing cost to purchaser's minor ordering cost for item i. (i.e. $\beta i = Si / Ai$)

N* Economic number of purchase orders in a year when all the items are ordered whenever a purchaser order is placed.

To simplify the calculation, we make following assumption

(1) $Rvi = Rpi$ (= R say) for all i

(2) αi is same for all i (and equal to α)

(3) βi is same for all i (and equal to β)

SOLUTION METHODOLOGY

We will customize the heuristic procedure given by Goyal(1974(b)) for our purpose. For explaining the heuristic, assume Hi as stock holding cost per unit per year, Y as major ordering cost, Yi as minor order placing cost for item i, N as frequency of purchase orders and Ni as the frequency of replenishment for the item i in a planning period, say 1 year. Define Ki = N / Ni as relative ordering frequency of the item i. clearly K values will be integers.

Objective is to minimize the total relevant cost (or total variable cost) of purchaser in this case. The total variable cost will be given by

$$C(N) = N[Y + \sum_{i=1}^{n} (Yi / ki)] + \frac{1}{2N} \sum_{i=1}^{n} (HiDiKi)$$

Following steps are performed to find out the optimum ordering policy

(1) Calculate $Ri = DiHi / Yi$.

(2) Assume arbitrary values for the relative ordering frequencies of the items. For the first set of computations, the relative ordering frequency for each item can be assumed as 1, and this initial policy provides the combination (1,1,1.....1) simply denoted as (Ki0)

(3) Take the first term in the list and determine the K_{11} values by comparing the ratio $B_{11}/ A_{11}R_1$ with the values given in the table 4.1 below. The B and A values can be obtained as per following equations

$$Ae1 = \sum_{i=1}^{c-1}(Yi/Ki) + \sum_{i=c+1}^{n}(Yi/Ki) + Y$$

and

$$Be1 = \sum_{i=1}^{c-1}(HiDiKi) + \sum_{i=c+1}^{n}(HiDiKi)$$

The new combination is (k_{11},1,1.....,1). Now move to the second item and determine $K_{21}$by evaluating the ratio B_{21} / $A_{21}R_2$) and comparing the values given in table 4.1. Similarly obtain the values K_{i1} for i = 1,2..., n.

(4) Applying step (3) to (K_{i1}) and obtain (K_{i2}) for i = 1,2,.....,n. The optimum policy is obtained in the (j + 1)th set of computation if

$$(K_{i(j+1)}) = (Kij) \text{ for all i;}$$

$$= (Ki^*)$$

convergence is normally very rapid (within fifth set of computation). No further improvement is possible in the policy which will be derived from (Ki*)

(5) The optimum policy can be obtained as follows

(a) Optimum number of purchase orders per year is given by

$$N^* = \sqrt{(\sum_{i=1}^{n} HiDiKi^*) \Big/ 2(Y + \sum_{i=1}^{n} Yi / Ki^*)}$$

(b) Optimum number of replenishment for the item i can be obtained by evaluating (N* / Ki*) for each item.

(c) The optimum order quantity for the item i is given by

$$Qi^* = DiKi^* / N^*.$$

(d) The total cost will be given as

$$C(N) = N^* (Y + \sum_{i=1}^{n} Yi / Ki^*) + \frac{1}{2N^*} \sum_{i=1}^{n} HiDiKi^*$$

Table 4.1

Calculation of K values

K values	K=1	K=2	K=3	K =4	K =5	K=6	K=7	K=8	K=9	K=10
lower bound	0	2	6	12	20	30	42	56	72	90
upper bound	2	6	12	20	30	42	56	72	90	110

Hence the optimum ordering (or manufacturing)policy can be find out for a number of items.

4.5 SOLUTION OF SINGLE SUPPLIER - SINGLE PURCHASER PROBLEM

Now we analyze the problem of single supplier single purchaser in case of multi-item inventory system. The solution procedure depend upon the type of coordination that exist between the two parties. As discussed in chapter 2, our objective, as in the case of single item, is to adopt the policy which minimizes the total cost of purchaser and supplier together. We will examine the Independent Optimization (IO), Joint Economic Lot Size Modeling (JELS), Individual Responsible And Rational Decision (IRRD) And System Approach (SA) concepts in this case.

A comparison of these approaches, using a numerical example has been carried out which indicate the superiority of SA over other concepts

4.5.1 INDEPENDENT OPTIMIZATION (IO)

Two possibilities exists

WHEN SUPPLIER OPTIMIZES

Here supplier (vendor) is given the power to decide the manufacturing policy (and ıce the ordering policy of purchaser) of the items under consideration. The supplier in s case determine the policy, which instead of minimizing joint total relevant cost ΓRC), minimizes the relevant cost of supplier leaving most of the time the purchaser at a eat disadvantage. The heuristic discussed above can be used in this case after doing llowing cost adjustments.

major manufacturing set up cost (Y) = $S + M$

minor manufacturing set up cost (Yi) = $Si + Mi$

inventory carrying cost (Hi) = $RvCvi$

2) WHEN PURCHASER OPTIMIZES

Here purchaser is given the power to decide the ordering policy (and hence the manufacturing policy of supplier) of the items under consideration. Purchaser will use his cost structure to decide about the policy, in isolation. The following costs will be used to determine the optimum policy for purchaser.

major ordering (or manufacturing) cost (Y) = A

minor ordering (or manufacturing) cost (Yi) = Ai

inventory carrying cost (Hi) = $RpCpi$

4.5.2 JOINT ECONOMIC LOT SIZE MODEL (JELS)

The basic philosophy of the JELS is to minimize the joint total relevant cost (JTRC) of both supplier and purchaser. Hence it take into consideration the total relevant cost of ordering as well as the holding of inventory, For the use of the heuristic discussed above, following cost should be used

major manufacturing cost (Y) = $A + M + S$

minor manufacturing cost (Yi) = Ai + Mi + Si

inventory carrying charge (Hi) = RpCpi + RvCvi

.3 INDIVIDUALLY RESPONSIBLE AND RATIONAL DECISION (IRRD) 'PROACH

This approach, as discussed in chapter 2 is close to IO (when purchaser optimizes). ie supplier lowers the price in the response of purchaser's decision to pay the order ndling and processing cost incurred by him each time when purchaser orders. The llowing costs are used to find the optimum ordering policy.

major ordering (or manufacturing) cost (Y) = A + S

minor ordering (or manufacturing) cost (Yi) = Ai + Si

inventory carrying cost (Hi) = RpC'pi

Iere C'pi is the new price of item i, and it is given by

C'pi = Cpi - Cri

Where Cri is the reduction in the price of item i, and given as

$$Cri = SN^* \Big/ \sum_{i=1}^{n} Di + \frac{SiNi^*}{Di}$$

Here N* = Economic number of purchase orders in a year when all the items are ordered whenever a purchase order is made in case of IO (when purchaser optimizes).

and Ni* = Frequency of replenishment for the item i in case of IO (when purchaser optimizes).

4.5.4 SYSTEM APPROACH (SA)

As discussed in chapter 2, SA is near to JELS approach philosophically because it optimizes the JTRC. The following costs are used to determine the optimum schedule

major manufacturing cost (Y) $= A + M + S$

minor manufacturing cost (Yi) $= Ai + Mi + Si$

inventory carrying charge (Hi) $= RpC''pi + RvC''vi$

ere $C''vi$ = New cost of production of item i ,after adopting SA

and $C''pi$ = New selling price of item i ($C''pi = \alpha C''vi$)

/i will be calculated as

$$C''vi = MN^* \Big/ \sum_{1}^{n} Di + MiNi^* / Di$$

Here N* and Ni* are as defined above.

5.5 AN EXAMPLE

Now we consider one example, as given in Goyal (1974(a)) with minor ıodifications (Table 4.2 shows the data) and find the optimum frequency per year for all ıe items and associated costs under different type of coordination (i.e. IO, JELS, IRRD, ;A). Tables 4.3 to 4.7 show the results. Percent saving in each case has also shown in Table 4.2

TABLE 4.2

PROBLEM DATA

major manuf. cost = 450 Rs.

major ordering cost = 100 Rs.

$Rp = Rv = 0.2$ Rs./Rs./unit

$\mu = 1.25$

$\beta i = 1.0$ for all i

item no..	demand per year in units	minor manuf. cost (Rs.)	minor ordering cost (Rs.)	cost of prod./unit (Rs.)
1	10000	80	20	10.0
2	12000	50	15	5.0
3	9000	100	40	10.0
4	10000	100	25	7.5
5	8000	70	10	6.25
6	6000	60	15	5.0
7	20000	120	30	2.5
8	12000	110	20	3.75
9	10000	100	20	4.0
10	5000	100	25	6.5
11	5000	80	15	5.0
12	10000	120	40	3.75
13	1500	30	5	6.0
14	2500	50	10	5.0
15	2000	70	15	8.0
16	4000	100	25	5.0
17	2000	80	10	7.5
18	350	40	20	10.0
19	600	20	10	2.5
20	1000	70	20	3.0

RESULTS

Results of this problem have been summarized in following tables.

TABLE 4.3

OPTIMUM POLICY PER YEAR

model type	optimum number of orders per year	optimum combination for K
IO (supp. opt)	6.00	(1,1,1,1,1,1,1,1,1,1, 1,1,1,1,1,2,2,3,3,3)
IO (purch. opt.)	15.81	(1,1,1,1,1,1,1,1,1,1, 1,2,1,1,1,2,1,3,4,4)
JELS	8.09	(1,1,1,1,1,1,1,1,1,1, 1,1,1,1,1,2,1,4,4,4)
IRRD	11.13	(1,1,1,1,1,1,1,1,1,1, 1,2,1,1,1,2,1,3,4,4)
SA	8.25	(1,1,1,1,1,1,1,1,1,1, 1,1,1,1,2,2,2,3,3,3)

TABLE 4.4

TOTAL RELEVANT COST OF SUPPLIER

model type	maj. man. set up cost	min. man. set up cost	maj. order handling cost	min order handling cost	inventory carrying cost	**total relevant cost**
IO (supp. opt)	2703	8250	600	2037	13591	**27183**
IO (pur. opt)	7116	21282	1581	5086	5334	**40399**
JELS	3643	11442	809	2786	9900	**28580**
IRRD	5012	14991	0	0	7572	**27575**
SA	0	0	0	0	9724	**9724**

TABLE 4.5

TOTAL RELEVANT COST OF PURCHASER

model type	maj ord -ering cost	min ord -ering cost	invent. carrying cost	**total relevant cost**
IO(sup.opt)	600	2037	16989	**19626**
IO(pur.opt)	1581	5086	6668	**13335**
JELS	809	2786	12376	**15971**
IRRD	2228	7166	9394	**18788**
SA	5358	16429	12064	**33851**

TABLE 4.6

SUMMARY

MODEL TYPE	JTRC (Rs.)	PERCENT SAVING
IO (supp. optimize)	46,809	12.8
IO (purch. optimize)	53,734	0.0
JELS	44,551	17.0
IRRD	46,363	13.7
SA	**43,575**	**18.9**

RESULT ANALYSIS

Table 4.3 gives the optimum policy under different models used. The optimum numbers of orders per year and combination of the K values varies significantly. Looking through the table one can conclude that JELS and SA values are in close approximation. This is because of the fact the both optimizes the joint total relevant cost unlike IO and IRRD,which optimizes the purchaser's total relevant cost.

Total relevant cost of vendor and purchaser are tabulated in table 4.4 and table 4.5 respectively, while saving under different model is given in table 4.6

The additional cost on purchaser due to adoption of SA (i.e. manufacturing setup and order handling and processing cost) is Rs.20,516, while the total relevant cost of vendor decreases by an amount of Rs.30,675. Hence there is net saving of Rs.10,159 under model SA. This saving should be shared by both the parties in some equitable fashion.

4.5.6 PARAMETRIC ANALYSIS

The variation in joint total relevant cost under basic models as discussed in this chapter, with respect to parameters is discussed now

(1)Varying β value

.βi denotes the ratio of vendor's minor order handling and processing cost to purchaser's ordering cost for an item i, and we assume for convenience that it is same for all items, and we take it as β. When β is zero, the results of IRRD approach coincide with IO (when purchaser optimizes) because of no cost to be transferred to purchaser side and hence no change in Cp value. But the larger the value of β, larger will be the reduction in Cp value and hence larger saving under IRRD. The saving under SA will be in same fashion as that of IRRD because of fixed manufacturing setup cost and hence reduction in Cv value. Figure 4.1 shows the variation of joint total relevant cost (JTRC) under basic

models with respect to β. Clearly SA perform better than any other model. As β increases, IRRD model approaches towards SA model because of more effectiveness of order handling and processing cost of vendor than manufacturing set up cost.

(2) Varying α value

α represents the profit margin and we have assumed that it is same for all the items. Higher α value implies that Cp is sufficiently larger as compared to Cv. When we adopt SA model under fixed profit margin policy (i.e. α is fixed), a small decrease in Cv value will result in marginal decrease in Cp. Due to lower value of Cp and Cv, joint total relevant cost under SA model will be less as compared to any other model. Figure 4.2 presents this interpretation graphically. JTRC under SA model is less than JELS, which in turn is lower than IRRD which is ultimately lower than IO (when purchaser optimizes) for any value of Cp/Cv. Because of fixed profit margin policy, variation in the JTRC is linear.

(3) Varying inventory carrying charge (R).

We assume inventory carrying charge (R) to be the same for all the items for connivance. Inventory carrying cost of vendor depends on RCv and the purchaser will depend on RCp. Lower value of R along with lower values of Cp and Cv are, thus required for inventory carrying cost to be lower. Under SA model because of lower values of Cp and Cv, there is saving in inventory carrying cost and hence saving in joint total relevant cost. Figure 4.3 shows the variation in JTRC under basic models with respect to R. It is clear from it that JELS behaves in close approximation with SA at lower values of R, but as R increases SA model shows marginal saving over JELS model.

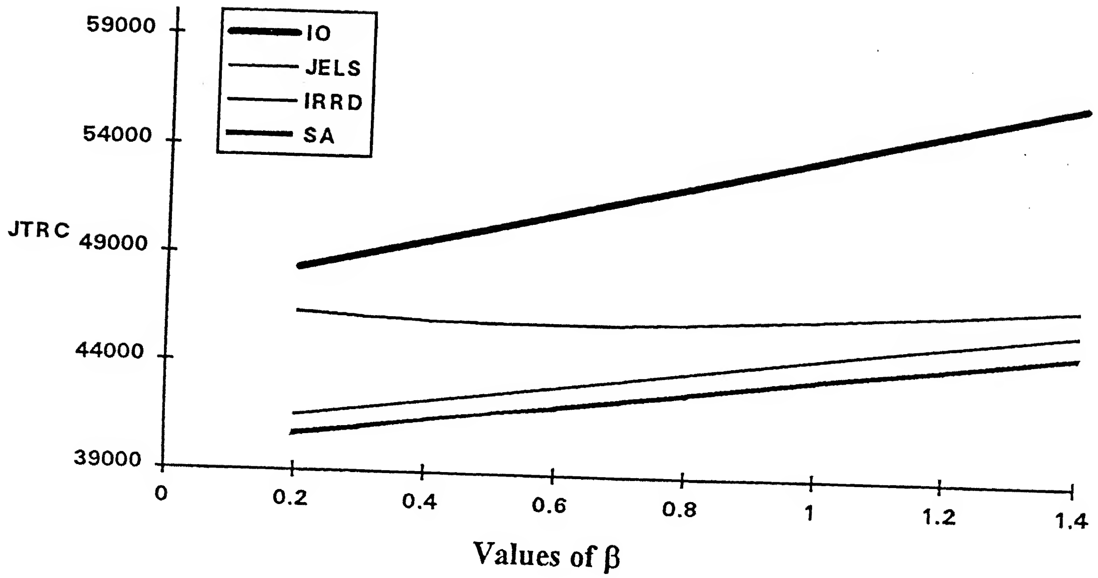
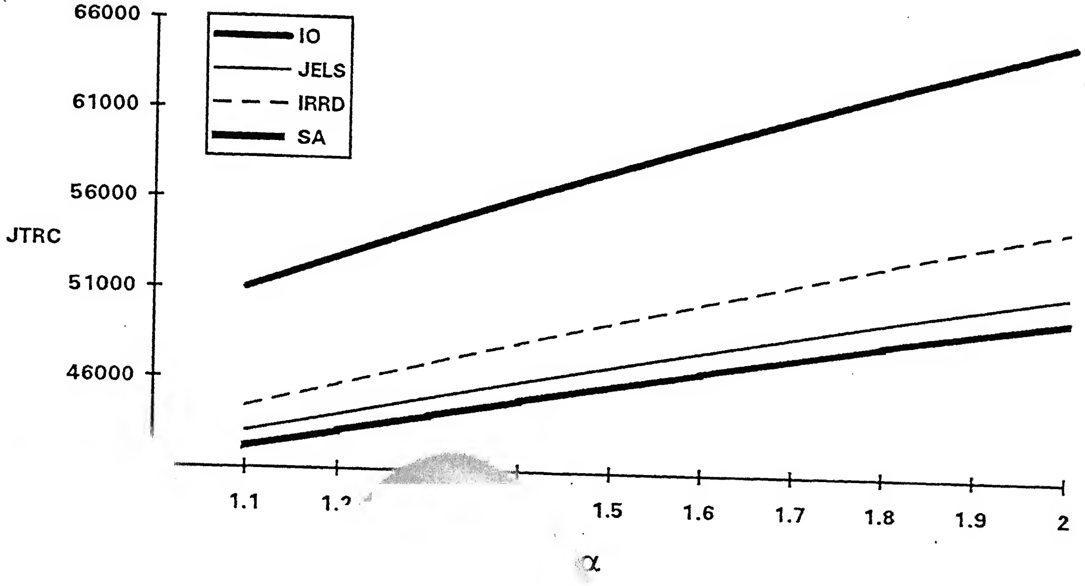


Fig 4.1
Effect of β on JTRC



JTRC

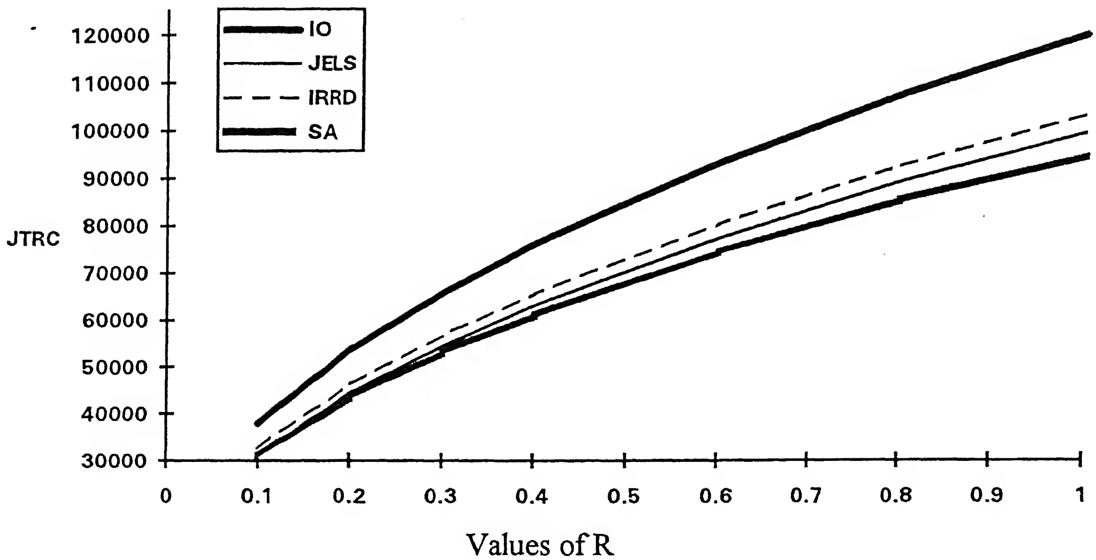


Fig 4.3
Effect of R on JTRC

CHAPTER 5

APPLICATION OF SYSTEM APPROACH IN JUST IN TIME PURCHASING ENVIRONMENT

5.1 INTRODUCTION

We now consider the relationship between a vendor and his purchaser under just in time (JIT) purchasing environment. This case is different than previous cases in the sense that purchaser require, inventory in small lots so as to minimize his inventory carrying costs. But vendor has to produce the ELS (may be the joint economic lot size) and if he transports the lot in small batches (as desired by purchaser), he will incur additional cost (inspection and shipping cost). Transportation cost, if paid by vendor than this situation is worst for him. On the other hand, purchaser has minimized his inventory carrying cost by adopting small lots. SA suits best in this situation because vendor is not bothered about handling and processing costs, hence he can process the whole lot in small sizes and purchaser can now afford the manufacturing set up costs because he is getting advantage due to lower carrying costs.

5.2 THE PROBLEM

We assume that a coordination exists between vendor and purchaser and they together decide about EOQ/ELS, which will be according to **JELS model.** With this Q, the JTRC will be minimum. Now purchaser wants that instead of supplying the whole lot, vendor should supply it in *uniform size sub-batches*. In doing so the inventory carrying cost of vendor will decrease substantially and now he can afford to pay the manufacturing set up costs incurred by the vendor. Vendor's inventory carrying costs will also reduce but

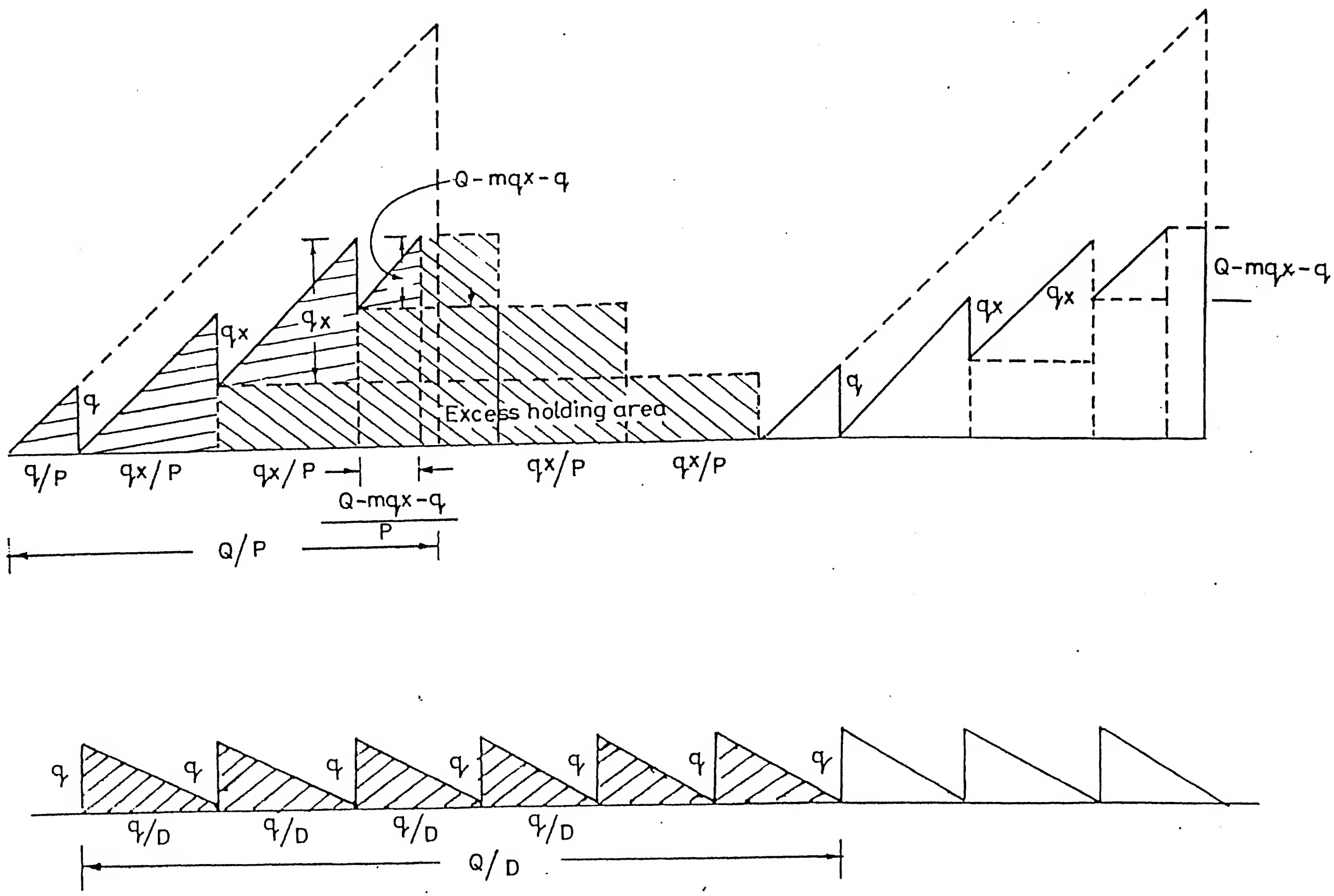


FIG 5.1 INVENTORY MODEL WITH SUPPLY IN UNIFORM SUB-BATCHES FOR FINISHED PRODUCT

transportation costs will increase. Both parties will be interested in finding out the subsize of shipment

Fig. 5.1 illustrates the proposed inventory situation. After the first sub-batch of q (uniform size) units is transported to the purchaser, then on receipt of this sub-batch the producer meets the demand continuously for next q/D units of time. During this period the vendor has already produced qx (as Pq/D = qx) units and it is ready for supply. As it is assumed here, the vendor would supply in uniform sub-batches of size q then even after supplying second sub-batch of q units, the vendor has to hold the remaining units. i.e. q (x -1) number of units. Since the purchaser receives only q units which he consumes at the rate D. Again during the next interval the supplier produces another round of qx units and supplies only q units to the purchaser and holds back remaining units. At this stage, due to q (x - 1) units held at the time of last supply, the total inventory left at the vendor's end will be 2q (x -1). This process continues till the vendor has produced Q. Now onwards, he will not produce and will supply from the stock he holds to the purchaser in equal lots of q, at the interval of q/D till it lasts. Thus, the stock is hold more on the vendor's side and less on the purchaser's side.

The relevant costs are annual cost of carrying inventory for the purchaser and vendor, the order cost for the purchaser, set up cost of vendor paid by purchaser, order handling and processing cost of vendor paid by purchaser and transportation cost. The horizon is assumed to be infinite and replenishment to be instantaneous.

5.3 MATHEMATICAL MODEL

As the order quantity is supplied to the purchaser in uniform sub-batch size of q, $nq = Q$.

$$\text{or} \qquad q = Q / n.$$

Referring to Fig. 5.1, the average inventory at the vendors end denoted by Xv, can be computed as given below :

$$Xv = [\frac{q}{2}\frac{q}{P} + \{\frac{qx}{2}\frac{q}{D} + q(x-1)\frac{q}{D}\} + ... + \{\frac{qx}{2}\frac{q}{D} + q(x-1)(m-1)\frac{q}{D}\}$$

$$+\{\frac{Q-(m-1)qx-q}{2}\}\{\frac{Q-(m-1)qx-q}{P}\}$$

$$+\{Q-(m-1)qx-q\}\{\frac{q}{D} - \frac{Q-(m-1)qx-q}{P}\}$$

$$+\{(1+2+3+..........+(n-m-1)q\frac{q}{D}\}] \, 1/_{Q/D}$$

or

$$Xv = [\frac{q^2}{2P} + \{\frac{q^2x}{2D} + q^2(x-1)\frac{1}{D\}} + ... + \{\frac{q^2x}{2D} + q^2(x-1)(m-1)\frac{1}{D}\}$$

$$+\frac{1}{2P}\{Q-(m-1)qx-q\}^2$$

$$+\{Q-(m-1)qx-q\}\{\frac{q}{D} - \frac{Q-(m-1)qx-q}{P}$$

$$+\frac{(n-m-1)(n-m)}{2}\frac{q^2}{D}]\frac{D}{Q}$$

On further simplifying the above equation and substituting the value of q (= Q/n), we get

$$Xv = \frac{Q}{2n^2}[\frac{1}{x} + m(2n-m-1) + (n-m-1)(n-m) - \frac{(n-1)^2}{x}] \quad ......(5.1)$$

Average annual inventory for the purchaser Xp will simply q/2

i.e. Xp = q/2 ......(5.2)

Now joint total relevant cost can be given as

JTRC = Inv. carrying cost for the vendor and the purchaser

+Avg. annual ordering cost of purchaser

+Avg. annual setup cost of the vendor

+Avg. annual order handling and processing cost of the vendor

+Avg. annual transpiration cost

or expressing mathematically

$$JTRC = RvCv'Xv + RpCp'Xp + D/Q(Sm+Sh+Sp+nSt) \qquad ......(5.3)$$

St is the cost of transporting one sub-batch from the vendor's end to the purchaser end. Cv′ and Cp′ are revised cost of product and selling price respectively after making necessary cost adjustment under application of SA.

assuming Rv=Rp=R and substituting the value of Xv and Xp from eqs.5.1 and 5.2 into eq.5.3, we get

$$JTRC = \frac{QRCv'}{2n^2}[\frac{1}{x} + m(2n-m-1) + (n-m-1)(n-m) - \frac{(n-1)^2}{x} + n\frac{Cp'}{Cv'}]$$

$$+D/Q[Sm+Sh+Sp+nSt]$$

On making following substitution

$$1/x + m(2n\text{-}m\text{-}1) + (n\text{-}m\text{-}1)(n\text{-}m) - (n\text{-}1)^2/x = C4 \qquad ....(5.4)$$

$$nCp'/Cv' = C5 \qquad ....(5.5)$$

JTRC then can be expressed as

$$JTRC = \frac{QRCv'}{2n^2}(C4 + C5) + \frac{D}{Q}(Sh + Sm + Sp + nSt) \qquad ....(5.6)$$

For a given value of m and n, the optimal lot size Q(n) is obtained by differentiating JTRC with respect to Q and setting the derivative equal to zero, this yields

$$Q(m,n) = \sqrt{\frac{2Dn^2(Sm + Sh + Sp + nSt)}{RCv'(C4 + C5)}} \qquad ....(5.7)$$

substituting from eq.5.7 to eq.5.6, we get

$$JTRC(m,n) = \sqrt{\frac{2RDCv'(Sm + Sh + Sp + nSt)(C4 + C5)}{n^2}} \qquad ....(5.8)$$

The optimal value of m and n i.e. m^* and n^* is obtained by evaluating JTRC(m,n) for their different positive integer values and choosing one that gives minimum JTRC.

Substituting the values of n and m corresponding to minimum JTRC in equations (5.4), (5.5) and (5.7), we can get the size of optimal order quantity.

RELATIONSHIP BETWEEN m AND n

There is a particular relationship that exist between m and n as can be seen from the following elaboration.

Since m represent the total number of sub-cycles where qx units are manufactured except q in first sub-cycle. In (m−1)th sub-cycle the vendor is in position to complete the production of q units fixed for that cycle

Thus

$$q+mqx > Q \geq q+(m-1)qx \qquad ....(5.9)$$

Since Q = nq, Therefor the above relationship can be rewritten as

$$q+mqx > nq \geq q+(m-1)qx$$

Or $$1+mx > n \geq 1+(m-1)x$$

Or $$mx > (n-1) \geq (m-1)x$$

Or $$mx > (n-1)$$

and $$(m-1)x \leq (n-1)$$

Or $$m > (m-1)/x \qquad ....(5.10)$$

and $$m \leq (n-1)/x + 1 \qquad ....(5.11)$$

Since m is an integer, therefor the following will definitely satisfy the relationship between m and n represented by eqs.(5.10) and (5.11).

$$m = [n-1/x+1] = [d]$$

Where [d] denotes smallest integer, greater than or equal to d.

CHAPTER 6

CONCLUSION AND SCOPE FOR THE FUTURE WORK

6.1 CONCLUSION

In the present work, the inventory problem of vendor and his purchaser(s) is taken into consideration in an integrated manner to minimize the joint total relevant cost. A new approach (named as system approach) has been developed and examined under single item and multi-item inventory system. A simple mathematical model has also been developed under JIT purchasing agreement between a vendor and a purchaser.

The application of SA under above mentioned inventory systems shows that SA is better than JELS and IRRD. Caparison of basic models indicate that superiority of IRRD and JELS (over each other) depend on the parameters value and neither is superior to another, like SA.

. The basic requirement for successful implementation of SA is the total coordination between vendor and purchaser(s). This coordination, has infect, started taking place because of increase in competition.

SA is best suited in the organization(s) where:

- A coordination between vendor and purchaser already exists and both parties are interested in working as a team in the matters related to the inventory control and want to reduce joint total relevant cost instead of individual's total relevant cost.
- Purchaser want that shipment to him should be in small sub-batches to reduce his inventory carrying costs.
- Numbers of the orders to be processed and handled by the vendor, annually is large.

- manufacturing setup cost, order handling and processing cost of vendor is high.
- Vendor adopts a high profit margin policy(i.e. high Cp/Cv ratio).
- Inventory carrying charge(Rv or Rp) is high.

6.2 SCOPE FOR THE FUTURE WORK

The present work, though gives a new approach to reduce the JTRC, is still far from complete in that, much more is to be done to evolve an operating policy for a practical situation. Following are some of the considerations which need further research:

- As demand fluctuations and lead time variations are not considered, model can be developed for such situation and can be studied.
- It should be noted that in this work, we have retained the JELS assumption that the purchaser's demand is constant regardless of the price reduction. The price reduction, we have proposed here need to be analyzed in term of their impact on the purchaser's demand.
- In practical situation, there exist a group of buyers and purchasers for single item/multi-item case. Application of SA under this environment will be more close to real world inventory problem.
- There is also scope to study the multi-stage, multi-product models in the light of frequent setups whether to allot machines exclusively for a particular product (for a particular customer) or different products.
- Demand substitution under multi-item case form another potential area for future research.

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